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Strong Dependence at the Long Run and the Cyclical Frequencies in the Specification of the US Unemployment Rate

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Abstract: This study deals with the presence of long range dependence at the long run and the cyclical frequencies in the specification of the US unemployment rate. We use a parametric procedure that permits us to test unit and fractional roots in raw time series. The results show that both the long run and the cyclical structures present a component of long memory behaviour. Additionally, the root at zero seems to be more important than the cyclical one, implying that shocks affecting the long run are more persistent than those affecting the cyclical part. The results are consistent with the empirical fact observed in many macroeconomic series that the long-term evolution is nonstationary, while the cyclical component is stationary and persistent.

Key words: Fractional Integration, Cyclical Behaviour, Long Memory

INTRODUCTION

Modelling macroeconomic time series is an area of research that has been widely investigated during the last twenty years. All the questions consist of finding the best approach to separate the business cycle, from seasonal and long run fluctuations. It should lead to an optimal decomposition of the raw series x_t into a seasonal movement (s_t), representing the persistent fluctuation of the series over the seasons, a trend movement (t_t), dealing with the long-run evolution of x_t, the business cycle movement (c_t) and an erratic component (u_t). Leaving out the seasonal part, the decomposition of economic series into a trend and a cycle remains an issue of considerable practical importance. Two methods have been widely employed. On the one hand, the unobserved component (UC) approach, introduced by Harvey [1] and Clark [2] and refined later by Harvey and Jaeger [3], implies a very smooth trend with a cycle that is large in amplitude and highly persistent. On the other hand, the approach of Beveridge and Nelson [4] implies that much of the variation of the series is attributable to variation in the trend, while the cycle is small and noisy. This conflict is theoretically solved by Morley *et al.* [5]. These authors show that since the two approaches are model-based, each leads to an ARIMA representation. Anyway, the economic theory in whatever format it comes has build-in the idea that cycles are stationary (if not in a wide sense, at least in a weak sense). That is, although there may be spillover of cyclical movements into the medium-long run, these are very small and nonstationary behaviour, if it exists, is due to the trend of the series The existing literature has usually concentrated on the long run behaviour of the series and unit roots have become a standard approach when modelling its behaviour. With respect to the cyclical part, stationary AR(2) processes have been widely employed.

The present study extends earlier work by adopting a modelling approach which, instead of considering exclusively the component affecting the long-run or zero frequency, also takes into account the cyclical structure. Using a large structure that involves simultaneously the zero and the cyclical frequencies, we can solve at least to some extent the problem of misspecification that might arise with respect to these two frequencies. We are able to show that our proposed method represents an appealing alternative to the increasingly popular ARIMA (ARFIMA) specifications found in the literature. It is also consistent with the widely adopted practice of modelling many economic series as two separate components, namely a secular or growth component and a cyclical one. The former, assumed in most cases to be nonstationary, is thought to be driven by growth factors, such as capital accumulation, population growth and technology improvements, whilst the latter, assumed to be covariance stationary, is generally associated with fundamental factors which are the primary cause of movements in the series.

MATERIALS AND METHODS

The methodology employed in this study has been used by Gil-Alana [6]. He analyses an extended version of Nelson and Plosser's dataset [7], which are fourteen US annual macroeconomic series. He concludes that all series may be specified in terms of fractional models with long memory with respect to both the zero and the cyclical frequencies. Of particular interest in that study are the results concerning the unemployment rate: It shows the smallest degree of integration at the zero frequency (0.84) and the highest one at the cyclical part

(0.11). In that study, however, the analysis reduces to the case of white noise disturbances and it does not allow for short run dynamics underlying the series. This study extends the results [6] by paying specific attention to the case of the unemployment rate, with the implications that the results might have in terms of theorizing, policy-making or forecasting.

The US Unemployment Rate: The stationary or nonstationary nature of the unemployment rate has been the study of numerous studies in recent years and it is a major preoccupation for macroeconomists and labour market economists. Recent contributions echoing this pessimistic conclusion are found in a study of US unemployment [8] and in general surveys of unemployment models [9, 10]. The problem that has increasingly become evident even in models using a large set of labour supply and institutional factors is that the models appear structurally (i.e. parameter) unstable. The argument we advance here is that this empirical problem is, in part, due to the inappropriate treatment of long and short run dynamics of unemployment in these models. On the one hand, the hysteresis approach to unemployment suggests that unemployment is a nonstationary highly persistent variable since fluctuations in the natural rate of unemployment are permanent. The econometric approach of modelling this behaviour is throughout the unit root model. On the other hand, the rejection of the unit root supports reversion to a natural rate, however, the slow reversion that is generally found is considered a form of hysteresis as well. Blanchard and Summers [11, 12] define hysteresis as: "a case where the degree of dependence is very high, where the sum of coefficients is close but not necessarily equal to one". This high persistence of shocks is a feature, among others, of "insider" models, [13], or of models in which fixed and sunk costs make current unemployment a function of past labour demand [14, 15]. The standard approach of looking at this problem, through the classical unit root tests, [16, 17], has two important limitations. First, it only considers integer values for the orders of integration: 1 in case of unit roots and 0 for stationarity, but it does not allow for fractional alternatives. Also, these methods have very low power in the context of fractional alternatives [18, 19]. Besides, these methods do not take into account the possible cyclical structure underlying the series and are therefore inadequate to describe the pattern observed in the unemployment rates.

In this study, we solve the two above-mentioned problems by means of a procedure that permits us to consider fractional orders of integration and cyclical structures. By using fractional orders of integration at these two frequencies, we allow for a much richer degree of flexibility in the dynamic behaviour of the series. Thus, if the order of integration at any frequency (denoted by d) is higher than 0 but smaller than 0.5, the series is covariance stationary, though shocks will take longer time to disappear than in the case of d = 0. In the latter case, the series is said to be "short memory" and shocks will disappear fairly soon, according to an exponential decay, if, for example, the observations are autoregressive. On the other hand, if d > 0, the series is said to be "long memory" and the decay is hyperbolic. The testing procedure described below permits us to consider unit and fractional orders of integration and it has standard null and local limit distributions. This is a distinguishing feature of the tests compared with other methods for testing, for example, unit roots, where the limit distribution is non-standard, in the sense that the critical vales have to be calculated numerically on a case by case simulation study. Moreover, this standard limit distribution holds independently of the inclusion or non-inclusion of deterministic components and thus, it is unaffected by the inclusion of intercepts or linear time trends.

The Statistical Model: We assume that $\{x_t, t = 1, 2, \ldots, T\}$ is the time series we observe, which is generated by the model:

$$(1 - L)^{d_1} (1 - 2\cos w L + L^2)^{d_2} x_t = u_t, t = 1, 2, \ldots, \quad (1)$$

with $x_t = 0$ for $t \leq 0$ and where L is the lag operator ($Lx_t = x_{t-1}$), w is a given real number, u_t is I(0), defined as a covariance stationary process with spectral density function that is positive and finite at any frequency on the spectrum. and where d_1 and d_2 can be real numbers. Let us first consider the case of $d_2 = 0$. Then, if $d_1 > 0$, the process is said to be long memory at the long run or zero frequency, also termed 'strong dependent' and so-named because of the strong association between observations widely separated in time. Note that the first polynomial in equation (1) can be expressed in terms of its Binomial expansion, such that for all real d_1,

$$(1 - L)^{d_1} = \sum_{j=0}^{\infty} \binom{d_1}{j} (-1)^j L^j = 1 - d_1 L + \frac{d_1 (d_1 - 1)}{2} L^2 - \ldots . \quad (2)$$

This type of process was initially introduced by Granger [20, 21] and Hosking [22] and it was theoretically justified in terms of aggregation [23-27] and in terms of the duration of shocks [28]. The differencing parameter d_1 plays a crucial role from both economic and statistical viewpoints. Thus, if $d_1 \in (0, 0.5)$, the series is covariance stationary and mean-reverting, with the effect of the shocks disappearing in the long run; if $d_1 \in [0.5, 1)$, the series is no longer stationary but it is still mean-reverting, while $d_1 \geq 1$ means nonstationarity and non-mean-reversion. It is therefore crucial to examine if d_1 is smaller than or equal to or higher than 1. Thus, for example, if $d_1 < 1$, there exists less need for policy action than if $d_1 \geq 1$ since the series will return to its original level sometime in the future. On the contrary, if $d_1 \geq 1$, shocks will be permanent and strong policy actions will be required to bring the variable back to its original long term projection.

Let us now consider the case of $d_1 = 0$ and $d_2 > 0$. The process is then said to be long memory at the cyclical part. It was examined by Gray et al. [29, 30] and they showed that the series is stationary if $|\cos w| < 1$ and $d < 0.50$ or if $|\cos w| = 1$ and $d < 0.25$. They also showed that the second polynomial in (1) can be expressed in terms of the Gegenbauer polynomial C_{j,d_2}, such that, calling $\mu = \cos w$,

$$(1 - 2\mu L + L^2)^{-d_2} = \sum_{j=0}^{\infty} C_{j,d_2}(\mu) L^j, \qquad (3)$$

for all $d_2 \neq 0$, where:

$$C_{j,d_2}(\mu) = \sum_{k=0}^{[j/2]} \frac{(-1)^k (d_2)_{j-k} (2\mu)^{j-2k}}{k!(j-2k)!}; (d_2)_j = \frac{\Gamma(d_2 + j)}{\Gamma(d_2)},$$

where, $\Gamma(x)$ represents the Gamma function and a truncation will be required in equation (3) to make the polynomial operational. Of particular interest is the case of $d_2 = 1$. Then, we say that the process contains a unit root cycle and its performance in the context of macroeconomic time series was examined, for example, by Bierens [31]. Unit root cycles have also been examined in other studies [32-36]. The economic implications here are similar to the previous case of long memory at the zero frequency. Thus, if $d_2 < 1$, shocks affecting the cyclical part will be mean reverting, while $d_2 \geq 1$ will imply persistence of the shocks forever. We next describe a version of a testing procedure, [37] that permits us to simultaneously consider the roots at zero and the cyclical frequencies.

The Testing Procedure: Following Bhargava [38], Schmidt and Phillips [39] on parameterization of unit root models, we consider a general model of form:

$$y_t = \beta' z_t + x_t, \quad t = 1, 2, \ldots \qquad (4)$$

where, y_t is a given raw time series; z_t is a (kx1) vector of exogenous variables; β is a (kx1) vector of unknown parameters; and the regression errors x_t are of form as in equation (1). Robinson [37] proposes a Lagrange Multiplier (LM) test of:

$$H_o: \ d \equiv (d_1, d_2)' = (d_{1o}, d_{2o})' \equiv d_o \qquad (5)$$

in a model given by the equations (1) and (4).

Clearly, d_{1o} corresponds to the order of integration at the long run or zero frequency, while d_{2o} refers to the degree of integration affecting the cyclical part. Additionally, we can take $w = w_r = 2\pi/r$, $r = 2, \ldots, T/2$, where r means the number of periods required to complete the whole cycle. Note that if $r = 1$, the cyclical part reduces to an I(d) process, with the singularity restricted exclusively to the long run or zero frequency. Based on H_o (5), the differenced series is given by:

$$\hat{u}_t = (1 - L)^{d_{1o}} (1 - 2\cos w L + L^2)^{d_{2o}} y_t - \hat{\beta}' s_t, \qquad (6)$$

$$\hat{\beta} = \left(\sum_{t=1}^{T} s_t s_t' \right)^{-1} \sum_{t=1}^{T} s_t (1-L)^{d_{1o}} (1 - 2\cos w L + L^2)^{d_{2o}} y_t,$$

$$s_t = (1 - L)^{d_{1o}} (1 - 2\cos w L + L^2)^{d_{2o}} z_t,$$

and it is assumed to have spectral density given by:

$$f(\lambda;\tau) = \frac{\sigma^2}{2\pi} g(\lambda;\tau), \qquad -\pi < \lambda \leq \pi,$$

where, the scalar σ^2 is known and g is a function of known form, which depends on frequency λ and the unknown (qx1) parameter vector τ. Unless g is a completely known function (e.g., $g \equiv 1$, as when u_t is white noise), we have to estimate the nuisance parameter τ, for example by $\hat{\tau} = \arg\min_{\tau \in T^*} \sigma^2(\tau)$, where T^* is a suitable subset of R^q Euclidean space and

$$\sigma^2(\tau) = \frac{2\pi}{T} \sum_{s=1}^{T-1} g(\lambda_s;\tau)^{-1} I_{\hat{u}}(\lambda_s), \text{ with}$$

$$I_{\hat{u}}(\lambda_s) = \left| (2\pi T)^{-1/2} \sum_{t=1}^{T} \hat{u}_t e^{i\lambda_s t} \right|^2; \lambda_s = \frac{2\pi s}{T}.$$

Note that the tests are purely parametric, requiring specific modelling assumptions to be made regarding the short memory specification of u_t. Thus, for example, if u_t is an AR process of form: $\phi(L)u_t = \varepsilon_t$, then, $g = |\phi(e^{i\lambda})|^{-2}$, with $\sigma^2 = V(\varepsilon_t)$, so that the AR coefficients are a function of τ.

The test statistic, which is derived via Lagrange Multiplier (LM) principle, adopts the form:

$$\hat{R} = \frac{T}{\hat{\sigma}^4} \hat{a}' \hat{A}^{-1} \hat{a}, \qquad (7)$$

where, T is the sample size and

$$\hat{a} = \frac{-2\pi}{T} \sum_s^{*} \psi(\lambda_s) g(\lambda_s;\hat{\tau})^{-1} I(\lambda_s);$$

$$\hat{\sigma}^2 = \sigma^2(\hat{\tau}) = \frac{2\pi}{T} \sum_{s=1}^{T-1} g(\lambda_s;\hat{\tau})^{-1} I(\lambda_s);$$

$$\hat{\varepsilon}(\lambda_s) = \frac{\partial}{\partial \tau} \log g(\lambda_s;\hat{\tau})$$

$$\hat{A} = \frac{2}{T}\left(\sum_s^{*} \psi(\lambda_s)\psi(\lambda_s)' - \sum_s^{*} \psi(\lambda_s)\hat{\varepsilon}(\lambda_s)' \left(\sum_s^{*} \hat{\varepsilon}(\lambda_s)\hat{\varepsilon}(\lambda_s)' \right)^{-1} \sum_s^{*} \hat{\varepsilon}(\lambda_s)\psi(\lambda_s)' \right)$$

$$\psi(\lambda_s)' = [\psi_1(\lambda_s), \psi_2(\lambda_s)];$$

$$\psi_1(\lambda_s) = \log\left|2\sin\frac{\lambda_s}{2}\right|;$$

$$\psi_2(\lambda_s) = \log\left|2\left(\cos\lambda_s - \cos w\right)\right|;$$

and the summation on * in the above expressions is over $\lambda \in M$ where $M = \{\lambda: -\pi < \lambda < \pi, \lambda \notin (\rho_k - \lambda_1, \rho_k + \lambda_1), k = 1, 2, \ldots, s\}$ such that ρ_k, $k = 1, 2, \ldots, s$ are the distinct poles of $\psi(\lambda)$ on $(-\pi, \pi]$.

Based on H_o (5), Robinson [37] established that, under certain regularity conditions

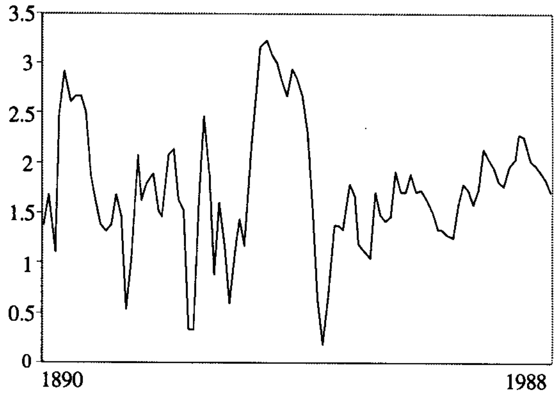
$$\hat{R} \to_d \chi_2^2, \qquad \text{as} \quad T \to \infty. \tag{8}$$

Thus and unlike other procedures, we are in a classical large-sample testing situation. A test of (5) will reject H_o against the alternative H_a: $d \neq d_o$ if $\hat{R} > \chi^2_{2,\alpha}$, where, Prob $(\chi_2^2 > \chi^2_{2,\alpha}) = \alpha$. Moreover the tests are efficient in the Pitman sense against local departures from the null, that is, if the tests are implemented against local departures of the form: H_a: $\theta = \delta T^{-1/2}$, for $\delta \neq 0$, the limit distribution is a $\chi_2^2(v)$, with a non-centrality parameter v, that is optimal under Gaussianity of u_t.

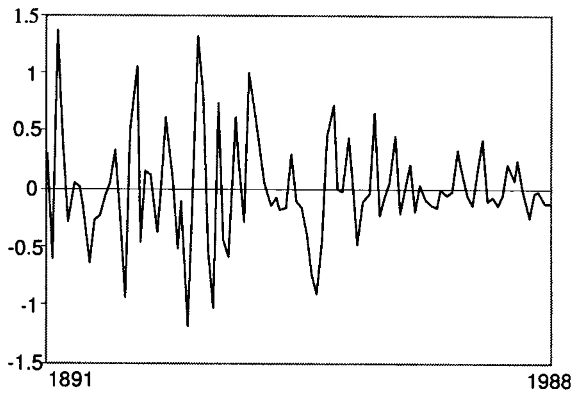
There exist other procedures for estimating and testing the fractionally differenced parameters, some of them also based on the likelihood function. Ooms [40] proposed tests based on seasonal fractional models. They are Wald tests and thus require efficient estimates of the fractional differencing parameters. He used a modified periodogram regression estimation procedure [41]. Also, Hosoya [42] established the limit theory for long memory processes with the singularities not restricted at the zero frequency and proposed a set of quasi log-likelihood statistics to be applied in raw time series. Unlike these methods, the tests of Robinson do not require estimation of the long memory parameters since the differenced series have short memory under the null. We believe that as in other standard large-sample testing situations, Wald and LR test statistics against fractional alternatives will have the same null and local limit theory as the LM tests of Robinson. With respect to the zero frequency, Sowell [43] employed essentially such a Wald testing procedure but it requires an efficient estimate of d_1 and while such estimates can be obtained, no closed-form formulae are available and so the LM procedure seems computationally more attractive.

RESULTS AND DISCUSSION

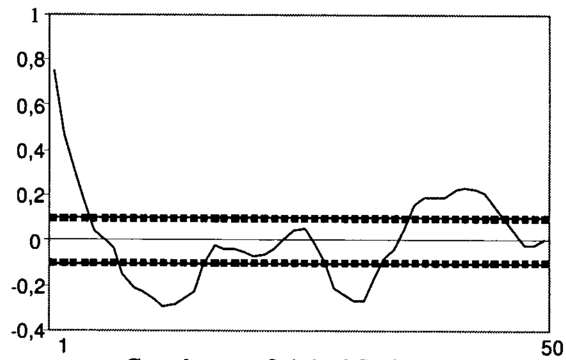
The dataset analysed is the US unemployment rate, annually, for the time period 1897-1988. We could have extended the series up to 2000. However, we have preferred to work with exactly the same data set as in Crato and Rothman [44] and Gil-Alana and Robinson [45] in order to get better comparisons with these two



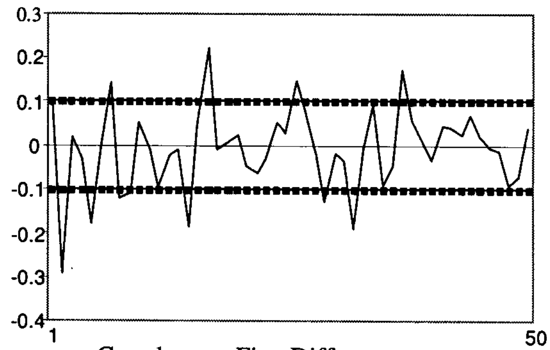
Original Time Series



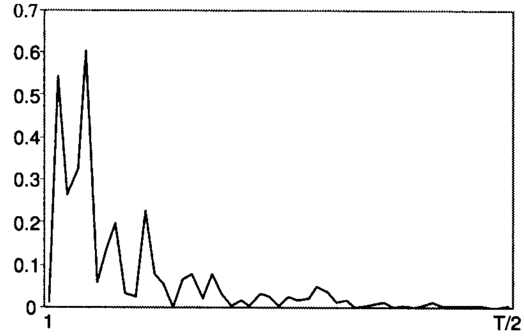
First Difference



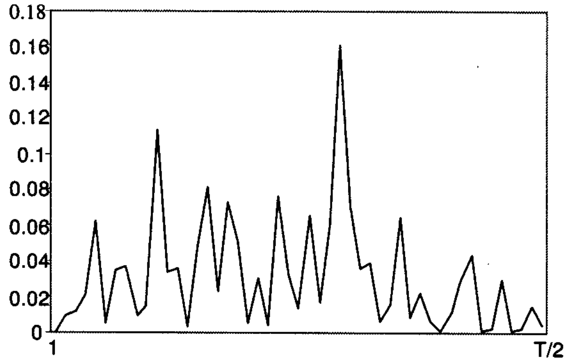
Correlogram Original Series



Correlogram First Difference



Periodogram Original Series



Periodogram First Difference

Fig. 1: US Unemployment Rate and its Differences with their Corresponding Correlograms and Periodograms

studies. The last eight observations will be discarded for forecasting purposes. This series is one of the fourteen macroeconomic series examined by Nelson and Plosser in the seminal study on unit roots [7]. It was also employed in [45] in the context of fractional models with a singularity restricted to the zero frequency. Nelson and Plosser, using tests of [16, 46] were unable to reject the existence of unit roots. However, [45] found that this variable could be specified in terms of an I(d) process with $d < 1$ and thus, showing mean reversion. We extend the latter approach to the case of fractional roots occurring simulataneously at zero and the cyclical frequencies.

Figure 1 displays plots of the original series and its first differences along with their corresponding correlograms and periodograms. The original series seems to be stationary, though the correlogram show significant values at some lags far away from zero, but also some apparent slow decay and/or cyclical oscillation, which could be indicative not only of fractional integration at zero but also of some cyclical dependence across the observations. Thus, it might be of interest to deeper examine this series in terms of a joint model for fractional integration at both the zero and the cyclical frequencies.

Denoting the US unemployment rate by y_t, we employ throughout the model given by the equations (1) and (4) with $z_t = (1,t)'$, $t \geq 1$, $(0, 0)'$ otherwise. Thus, under H_o (5), the model becomes:

$$y_t = \beta_0 + \beta_1 t + x_t, \qquad t = 1, 2, ... \qquad (9)$$

$$(1 - L)^{d_{1o}} (1 - 2\cos w L + L^2)^{d_{2o}} x_t = u_t, \quad t = 1, 2, ..., \qquad (10)$$

and if $d_{2o} = 0$, the model reduces to the case of long memory exclusively at the long run or zero frequency. We consider separately the cases of $\beta_0 = \beta_1 = 0$ a priori, (i.e., including no regressors in the undifferenced model (9)); β_0 unknown and $\beta_1 = 0$ a priori, (i.e., with an intercept) and β_0 and β_1 unknown (with an intercept and with a linear time trend) and assume that $w = w_r = 2\pi/r$, r indicating the number of time periods per cycle.

We computed the statistic $\hat{R}$ given by equation (7) for values d_{1o} and $d_{2o} = 0, (0.01), 2$ and $r = 2, ..., T/2$, assuming first that u_t is white noise. In other words, for each r, we compute the test statistic for all possible combinations of d_1 and d_2, with 0.01 increments. We do not report, however, the results for all statistics, though it was observed that the null hypothesis was rejected for all values of d_{1o} and d_{2o} if r was smaller than 4 or higher than 7, implying that if a cyclical component is present, its periodicity is constrained between these two years. This is consistent with the empirical findings that say that cycles have a duration constrained between 3 and 8 years [47-52].

Table 1 displays for each r (=5, 6 and 7), each u_t (white noise, AR(1) and AR(2)) and each z_t, (no regressors, an intercept and an intercept and a linear time trend), the values of d_1 and d_2 that produce the lowest statistics across the d's. Therefore, these estimates approximate to the maximum likelihood estimates of the fractional differencing parameters. We also display in the table the values of $\hat{R}$, the estimates of β_0 and β_1 and the AR parameters. Starting with r = 5, we see that the results substantially differ depending on the structure of the disturbances. Thus, if u_t is white noise, the order of integration at 0 (d_1) is 0.84 or 0.89, while d_2 ranges between 0.13 and 0.15. If u_t is AR(1), the results are identical for the three cases of no regressors, an intercept and a linear time trend, with $d_1 = 0.10$ and $d_2 = 0.11$. Finally, if u_t is AR(2), d_2 is in all cases equal to 0 and $d_1 = 0.07$ and 0.08. With some slight differences, very similar results are obtained for the cases of r = 6 and 7, the only exception being the case of r = 7 with AR(2) u_t. In such a case d_1 is strictly higher than 1, while d_2 ranges between 0.14 and 0.21. As a conclusion, we can summarize the results in this table by saying that they substantially change depending on the structure of the disturbances, in particular if they are Form autocorrelated or not.

Table 1: Selected Models that Produce the Lowest Statistics Across d_1 and d_2

r	u_t	z_t	d_1	d_2	$\hat{R}$	β_0	β_1	τ_1	τ_2
5	White noise	---	0.89	0.13	0.00056	---	---	---	---
		1	0.84	0.15	0.00023	0.38810*	---	---	---
		(1, t)'	0.84	0.15	0.00003	1.41923*	-0.00613	---	---
	AR (1)	---	0.10	0.11	0.00397	---	---	0.72835	---
		1	0.10	0.11	0.00437	0.02238*	---	0.72802	---
		(1, t)'	0.11	0.11	0.00497	0.03064*	-0.00138	0.72021	---
	AR (2)	---	0.07	0.00	0.27701	---	---	0.88929	-0.21818
		1	0.07	0.00	0.27756	0.01121*	---	0.88919	-0.21822
		(1, t)'	0.08	0.00	0.28689	0.01707*	-0.00104	0.88044	-0.21660
6	White noise	---	0.84	0.11	0.00195	---	---	---	---
		1	0.77	0.16	0.00104	1.36515*	---	---	---
		(1, t)'	0.77	0.16	0.00069	1.39952*	-0.00675	---	---
	AR (1)	---	0.04	0.11	0.00187	---	---	0.73293	---
		1	0.04	0.11	0.00191	0.00434	---	0.73291	---
		(1, t)'	0.04	0.11	0.00418	0.00552	-0.00053	0.73200	---
	AR (2)	---	0.03	0.00	1.01690	---	---	0.91683	-0.23116
		1	0.03	0.00	1.01719	0.00241	---	0.91683	-0.23117
		(1, t)'	0.03	0.00	1.04164	0.00334	-0.00043	0.91637	-0.23154
7	White noise	---	0.79	0.15	0.00102	---	---	---	---
		1	0.70	0.20	0.00128	1.34896*	---	---	---
		(1, t)'	0.69	0.20	0.00113	1.37295*	-0.00668	---	---
	AR (1)	---	0.01	0.10	0.75430	---	---	0.72994	---
		1	0.01	0.10	0.75428	0.00151	---	0.72994	---
		(1, t)'	0.01	0.10	0.78251	0.00185	-0.00015	0.78251	---
	AR (2)	---	1.09	0.14	0.00064	---	---	-0.10477	-0.39440
		1	1.06	0.21	0.25525	1.37550*	---	-0.18425	-0.35156
		(1, t)'	1.06	0.21	0.26716	1.40882*	0.00947	-0.18558	-0.34992

* significant coefficients at the 5% level

Table 2: Diagnostic tests for the selected models

R	No.	u_t	z_t	d_1	d_2	β_0	τ_1	τ_2	Diagnostics
5	1	White noise	Intercept	0.84	0.15	0.38810	---	---	A B C
	2	AR (1)	Intercept	0.10	0.11	0.02238	0.72802	---	B C
	3	AR (2)	No regr.	0.07	0.00	---	0.88929	-0.21818	B
6	4	White noise	Intercept	0.77	0.16	1.36515	---	---	A B C
	5	AR (1)	No regr.	0.04	0.11	---	0.73293	---	B C
	6	AR (2)	No regr.	0.03	0.00	---	0.91683	-0.23116	B
7	7	White noise	Intercept	0.70	0.20	1.34896	---	---	A B C
	8	AR (1)	No regr.	0.01	0.10	---	0.72994		B
	9	AR (2)	No regr.	1.09	0.14	---	-0.10477	-0.39440	A B C

A: Refers to Homoscedasticity, B: No autocorrelation C: Functional

Thus, if they are white noise, the order of integration at the long run or zero frequency is in all cases higher than 0.5 but smaller than 1, implying nonstationarity and mean reverting behaviour, while d_2 is constrained between 0.10 and 0.20. If u_t is AR, d_1 is smaller by about 0.50 compared with the case of white noise u_t, with d_1 being smaller than 0.20 in practically all cases. This reduction in the order of integration at the zero frequency may be explained by the fact that the AR parameters are Yule-Walker estimates and thus, though they entail roots that are automatically smaller than 1 in absolute value, they can be arbitrarily close to 1 and thus, they might be competing with d_1 in describing the nonstationary nature of the series at such a frequency. In the following table we try to be more specific about which might be the best model specification for this series and we start with the specification of the deterministic trends. We observed in Table 1 that the coefficients corresponding to the time trend were found to be insignificantly different from zero. Note that these estimates are all based on the null differenced model, which is assumed to be I(0) and thus, standard t-tests apply. With respect to the intercepts, they are significant in many cases. In those cases where the intercept is not significant, we choose the model with no regressors. Thus, we have nine potential models for the series, one for each r = (5, 6 and 7), with white noise, AR(1) and AR(2) disturbances. The last column of the table reports the results of several diagnostic tests carried out on the residuals. In particular, we perform tests of homoscedasticity, no serial correlation and functional form, using Microfit.

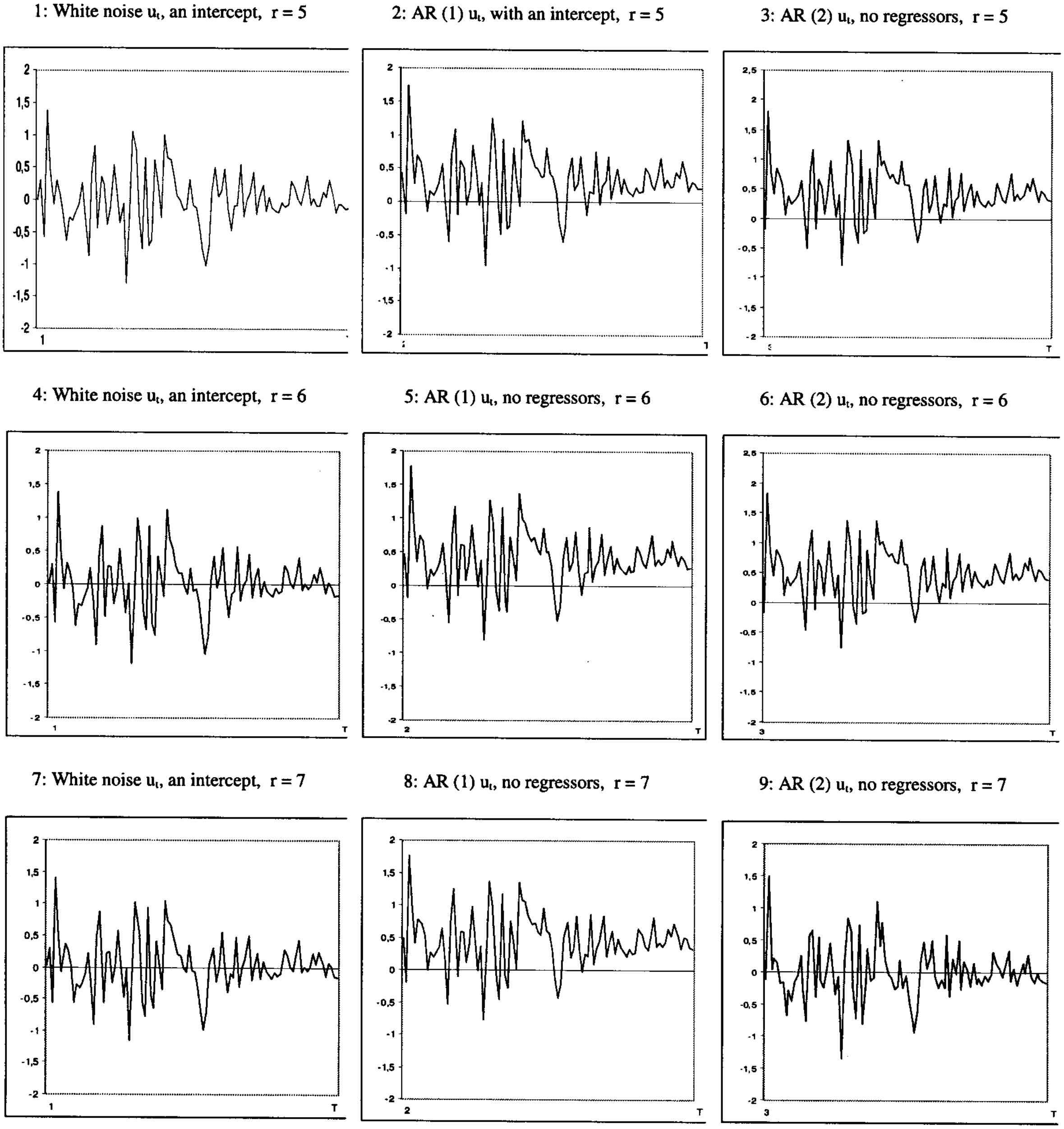


Fig. 2: Residuals from the Selected Models in Table 2

The results show that there are four models passing all the diagnostics: models 1, 4, 7 and 9. The first three of these models correspond to the case of white noise u_t, with an intercept and r = 5, 6 and 7 respectively, while model 9 refers to r = 7 with AR(2) u_t and no regressors. Figure 2 displays plots of the residuals of the selected models in Table 2. As expected from the diagnostics, models 1, 4, 7 and 9 show the closest residuals to white noise.

Figure 3 displays the first 50 impulse responses for the three models with white noise disturbances (1, 4 and 7). Model 9 is discouraged in view of the fact that d_1 is higher than 1, which is quite implausible, especially if we take into account that unemployment rate is a bounded variable [53]. We see in this figure that the three models present a very similar path, with a very slow (hyperbolic) decay. Thus, even 50 periods after the initial shock, more than 20% of its effect remains in the series. In Fig. 4 we concentrate on model 4 and disaggregate the impulse responses into the trend and the cyclical components. It is observed that most of the variation in the impulse responses is due to the long-term component. The same happens with respect to the other two models.

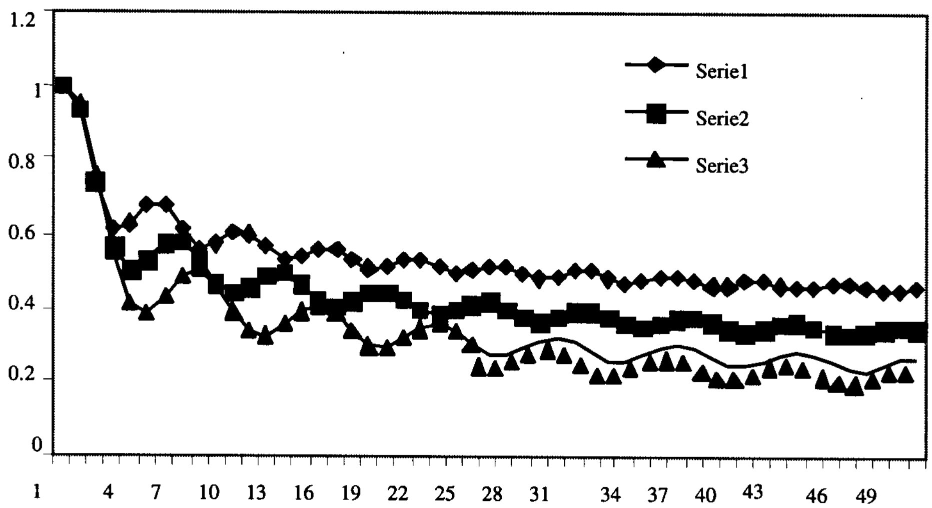


Fig. 3: Impulse Response Functions in Models 1, 4 and 7. Series 1 Refers to Model 1, Series 2 to Model 4 and series 3 to Model 7

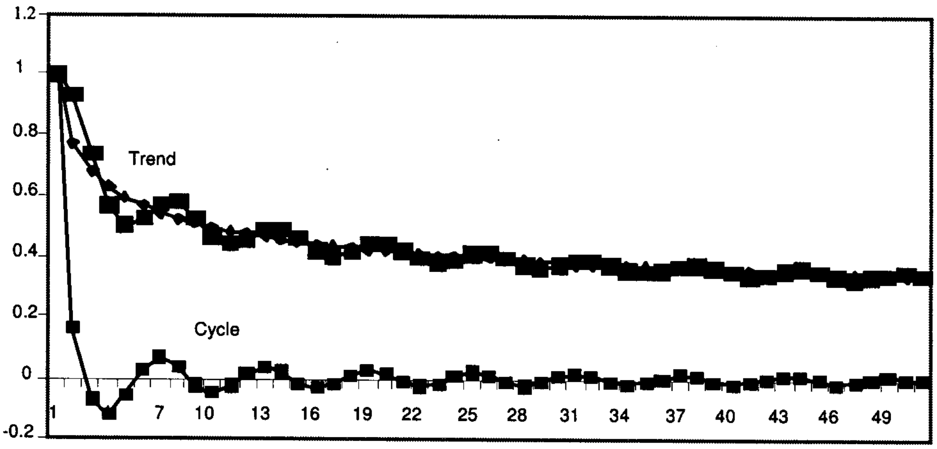


Fig. 4: Impulse Response Function in Model 4 Disaggregated for the Trend and the Cycle

Table 3: Selected Models Used in the Forecasting Exercise in Section 6

Number	Models employed in the forecasting exercise
Model 1	$y_t = 0.38810 + x_t; \quad (1\text{-}L)^{0.84}(1\text{-}2\cos w_5 L + L^2)^{0.15} x_t = \varepsilon_t$
Model 4	$y_t = 1.36515 + x_t; \quad (1\text{-}L)^{0.77}(1\text{-}2\cos w_6 + L^2)^{0.16} x_t = \varepsilon_t$
Model 7	$y_t = 1.34896 + x_t; \quad (1\text{-}L)^{0.70}(1\text{-}2\cos w_7 + L^2)^{0.20} x_t = \varepsilon_t$
ARMA (1, 1)	$y_t = 0.83058 + 0.53257 y_{t-1} + \varepsilon_t + 0.61857 \varepsilon_{t-1}$
ARIMA (0, 1, 2)	$(1 - L) y_t = -0.00694 + \varepsilon_t. + 0.17719 \varepsilon_{t-1} - 0.35817 \varepsilon_{t-2}$
ARIMA (1, 1, 2)	$(1 - L) y_t = -0.00662 + 0.27850 y_{t-1} + \varepsilon_t. - 0.06769 \varepsilon_{t-1} - 0.45037 \varepsilon_{t-2}$
ARFIMA (0 0.85, 0)	$y_t = 1.46257 + x_t; \quad (1\text{-}L)^{0.85} x_t = \varepsilon_t$

Forecasting Performance: Finally, we deal with the forecasting ability of the proposed models in comparison with other more classic representations for the US unemployment rate. In particular, we compare the fractional cyclical models proposed above with ARMA, ARIMA and ARFIMA models. We start by performing several unit root tests and use tests of [16, 46]. In both cases, we were unable to reject the null hypothesis of a unit root in most of the cases. However, it is well known that these tests have very low power against both AR and fractional departures. Thus, we try ARMA(p, q) and ARIMA(p, 1, q) models, with p and q smaller than or equal to 3. We use the Akaike (AIC) and Bayeian (BIC) information criteria. Starting with the ARMA case, both criteria lead to the same model, with p = q = 1. Using the ARIMA models, the AIC chooses an ARIMA(1, 1, 2), while the BIC an ARIMA(0, 1, 2). Finally, we also permit ARFIMA models and using a procedure of estimating by maximum likelihood [43], again with p, q $\leq$ 3, the selected model is an ARFIMA(0, 0.85, 0) with an intercept. In another study, the authors conclude that the best model specification is an ARFIMA(0, 1.01., 0) but they do not include deterministic trends [44]. Table 3 displays all the selected models, along with their parameter estimates.

We compare all these models in terms of their forecasting performance. Standard measures of forecast accuracy are the following: Theil's U, the mean absolute percentage error (MAPE), the mean-squared error (MSE), the root-mean-squared error (RMSE), the root-mean-percentage-squared error (RMPSE) and mean absolute deviation (MAD) [54]. Let x_t be the actual value in period t; f_t the forecast value in period t and n the number of periods used in the calculation. Then:

a. Theil's U: $\frac{\sqrt{\Sigma(x_t - f_t)^2}}{\sqrt{\Sigma(x_t - x_{t-1})^2}}$;

b. Mean absolute percentage error (MAPE): $\frac{\Sigma|(x_t - f_t)/x_t|}{n}$;

c. Mean squared error (MSE): $\frac{\Sigma(x_t - f_t)^2}{n}$;

d. Root-mean-percentage-squared error (RMSP): $\sqrt{\frac{\Sigma(x_t - f_t)^2/f_t}{n}}$;

e. Root-mean-squared error (RMSE): $\sqrt{\frac{\Sigma(x_t - f_t)^2}{n}}$;

f. Mean absolute deviation (MAD): $\frac{\Sigma|x_t - f_t|}{n}$.

The first type of evaluation criteria measures the spread or dispersion of the forecast value from its mean. The MAD belongs to this category. It measures the magnitude of the forecast errors. Its principal advantages are ease of interpretation and the fact that each error term is assigned the same weight. However, by using the absolute value of the error term, it ignores the importance of over or underestimation.

The second type of accuracy measure is based on the forecast error, which is the difference between the observation, x_t and the forecast, f_t. This category includes MSE, RMSE and RMSPE. MSE is simply the average of squared errors for all forecasts. It is suitable when more weight is to be given to big errors, but it has the drawback of being overly sensitive to a single large error. Further, just like MAD, it is not informative about whether a model is over- or under-estimating compared to the true values. RMSE is the square root of MSE and is used to preserve units. RMSPE differs from RMSE in that it evaluates the magnitude of the error by comparing it with the average size of the variable of interest. The main limitation of all these statistics is that they are absolute measures related to a specific series and hence do not allow comparisons across different time series and for different time intervals. By contrast, this is possible using the third type of accuracy measure, such as MAPE, which is based on the relative or percentage error. This is particularly useful when the units of measurement of x are relatively large. However, MAPE also fails to take over or under estimation into consideration.

Unlike the measures mentioned above, Theil's U is a relative measure, allowing comparisons with the naïve ($x_t = x_{t-1}$) or random walk model, where a U = 1 indicates that the naïve method is as good as the forecasting technique, whilst U < 1 means that the chosen forecasting method outperforms the naïve model. The smaller the U-statistic, the better the performance of the forecasting technique relative to the naïve alternative. Despite some attractive properties, the U-statistic has the disadvantage of not being as easily interpretable as MAPE; further, it does not have an upper bound and therefore is not robust to large values.

The seven selected time series models (fractional and cyclical differencing, FCD; fractional differencing, FD; and integer differencing, ID) were used to generate the 8-year-ahead out-of-sample forecasts. Each forecast value was calculated and compared with the actual value of the series. Then, the above six criteria were used to rank the forecasting ability of the proposed models. The ranking in terms of forecasting performance is given in Table 5

Computing forecasts within fractional (cyclical or non-cyclical) models is not trivial since the model cannot be written as a finite order ARMA model [55]. However, it can be sorted out by using the assumption $y_t = 0$, for $t \leq 0$ and the recursive equations, which are involved in the fractional processes. The forecasts for the zero and cyclical fractional models were obtained as follows. Consider, for example, Model 1:

$$\begin{aligned} y_t &= 0.38810 + x_t; \\ (1 - L)^{0.84}(1 - 2\cos w_5 L + L^2)^{0.15} x_t &= \varepsilon_t. \end{aligned} \qquad (11)$$

This model can be re-written as follows:

$$(1 - L)^{0.84}(1 - 2\cos w_5 L + L^2)^{0.15} y_t = 0.38810 w_t + \varepsilon_t, \qquad (12)$$

where, $w_t=(1-L)^{0.84}(1-2\cos w_5L+L^2)^{0.15}1_t$ and 1_t is the t^{th} element of a time series vector of 1s. Using now the expansions in equations (2) and (3) in the left-hand-side in (12)

$$\begin{aligned} &(1 - L)^{0.84}(1 - 2\cos w_5 L + L^2)^{0.15} y_t \\ &= (1 - \alpha_1 L - \alpha_2 L^2 - \ldots)(1 - \beta_1 L - \beta_2 L^2 - \ldots) y_t, \end{aligned} \qquad (13)$$

and thus, rearranging (12) and (13),

Table 4: One-period-ahead Forecast for Each of the Selected Models

Year	True value	Model 1	Model 4	Model 7	ARMA	$ARIMA^{BIC}$	$ARIMA^{AIC}$	ARFIMA
1981	2.028	1.966	2.002*	1.976	1.974	1.971	1.999	2.001
1982	2.272	1.947	2.010	2.019	1.943	1.982	1.967	2.216*
1983	2.261	2.178	2.185	2.218	2.243	2.296	2.273*	2.227
1984	2.014	2.198	2.018*	2.152	2.046	2.144	2.102	2.025
1985	1.974	2.017	1.944	1.908	1.884	1.997	1.920	1.979*
1986	1.945	2.013	1.946*	1.928	1.937	2.009	1.997	1.949
1987	1.824	1.951	1.840*	1.953	1.872	1.936	1.942	1.842
1988	1.704	1.810	1.867	1.865	1.772	1.820	1.821	1.731*

* The forecast that most approximate to the true value across the different models

Table 5: Overall Ranking of Forecasting Performance using Different Criteria

Model	Theil's U	MAPE	MSE	RMSD	RMSE	MAD
Model 1	7	6	7	7	7	7
Model 4	2	1	2	2	2	2
Model 7	4	3	4	4	4	6
ARMA	3	7	3	3	3	3
ARIMA	6	2	5	5	5	5
ARIMA	5	4	6	6	6	4
ARFIMA	1	5	1	1	1	1

$$y_t = (\gamma_1 L + \gamma_2 L^2 + \gamma_3 L^3) y_t + 0.38810 w_t + \varepsilon_t,$$

where, the γs were obtained throughout the combinations of the lags polynomial in the right-hand-side in equation (13). Then, it can be easily seen that:

$$\tilde{y}_{T+k|T} = (\gamma_1 L + \gamma_2 L^2 + \gamma_3 L^3) y_{T+k|T} + w_{T+k|T}.$$

Table 4 resumes the 1-period-ahead forecasts for each of the selected models. We see that for most of the years, Model 4 produces the best results, followed by the ARFIMA model. However, if we make the predictions in 1980 with an 8-year horizon, we see, in Table 5, that the best results are those corresponding to the ARFIMA model, i.e., with no cyclical components, though followed very close by Model 4 in all cases except for the MAPE criterion, where Model 4 seems to be again the best approach.

CONCLUSION

In this study we have presented a testing procedure that permits us to simultaneously consider unit and fractional roots at the long run and the cyclical frequencies in raw time series. The tests are very general and permit us to consider as particular cases of interest the situations of unit (or fractional) roots either at zero or the cyclical components. Unlike most of other procedures, they have standard null and local limit distributions and this standard behaviour holds independently of the inclusion or non-inclusion of deterministic trends and autocorrelated disturbances.

This is an unusual property compared with other methods involving nonstationary fractional structures, where the limit distribution have to be obtained numerically on a case by case simulation study. A Monte Carlo simulation work [6] shows that the tests described above perform relatively well even with small samples of an approximate size to the one used in this work.

The tests were applied to the unemployment rate in the US, using a series that is part of an extended version of Nelson and Plosser's dataset [7]. This series was also examined in [44, 45]. However, in these two studies, they just concentrate on the long run or zero frequency and do not pay any attention to the possible cyclical structure underlying the series. The results show that both the long run and the cyclical structures present a component of long memory behaviour. Additionally, the root at zero seems to be more important than the cyclical one, implying that shocks affecting the long run are more persistent than those affecting the cyclical part. They are also consistent with the empirical fact observed in many macroeconomic series that the long-term evolution is nonstationary, while the cyclical component is stationary and persistent.

Our model differs from other trend-cycle decomposition models, (unobservable components (UC), Beveridge and Nelson (BN)), mainly in the treatment of the cyclical part. Thus, the UC models consider the trend as a random walk, (or more generally an I(1)), process, while the cycle is described by a stationary ARMA(p, q) process. Harvey and Jaeger [3] suggest specifying p = 2, which allows the cycle to be periodic in the sense of having a peak in its spectral density function. In our model, we also allow for a peak at the spectrum, however, instead of using autoregressions, which produce abrupt changes in its asymptotic behaviour, we consider fractional models, which the corresponding smoothness associated to the limit behaviour across the orders of integration.

It would also be worthwhile proceeding to get point estimates of the fractional differencing parameters in this context of trends and cyclical models. For the trending component the literature is extent, [56-61]. For the cyclical part, some attempts have been made in [62, 63]. However, the goal of this study is to show that a fractional model with the roots simultaneously occurring at zero and the cyclical frequencies can be a credible alternative when modelling the US unemployment rate. In that respect, the results presented in this work leads us to some unambiguous conclusions, with the periodicity constrained between 4 and 7 years and the order of integration being higher at the zero frequency than at the cyclical part.

A potential drawback of the present work is that it is based on an univariate model, with the limitation that it imposes in terms of theorizing, policy-making or forecasting. Theoretical models and policy-making involve the relationships between many variables and the forecast performance can be improved through the use of many variables (e.g., factor based forecasts based on data involving hundreds of time series beat univariate forecasts [64]). However, the univariate work has relevance in the context of business cycles, firstly because different time series may have different amplitudes and different orders of integration and there is not yet theoretical econometric models that permit us to examine cyclical fractional models in a multivariate framework. In that respect, the present study can be considered as a preliminary step in the analysis of business cycles from a different time series perspective. Finally, the issue of data mining is another worry for economists when looking at time series models. There are so many possible models that may be relevant and so many modelling choices that econometricians are almost sure to find something purely by data mining. For this reason, sequential testing and other procedures based on information criteria are widely distrusted and model averaging methods have become very popular. Thus, it might also be worthwhile to broaden the class of models under consideration and address the data mining problem, along with other issues (e.g., structural breaks) using averaging approaches. Work in all these directions is now under progress.

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