Reed Solomon Coding for Frequency Hopping Spread Spectrum in Jamming Environment

Othman Sidek and Abid Yahya
Collaborative MicroElectronic Design Excellence Center
Universiti Sains Malaysia, Engineering Campus, Nibong Tebal, Penang, Malaysia

Abstract: Frequency hopping spread spectrum with noncoherent M-ary frequency shift keying system is considered. In this study Reed Solomon codes RS (255, 223) are employed in a standard error correction role to provide some Anti-Jamming (AJ) capability with 8-bit symbols. Noise has been added to RS encoded data, which is demodulated at RS decoder perfectly. Matlab simulations have been implemented for numerical results. Graphical results have been plotted between the probability of an erasure versus ρ using RS (255, 223) code for 1 and 2 kHz frequencies. The simulation results have proved that system having RS (255, 223) code with M = 32 require substantially less signal to jammer power ratio to achieve the same bit error probability.

Key words: M-ary frequency shift keying (MFSK), anti-jamming, error control, probability

INTRODUCTION

Frequency Hopping Spread Spectrum (FHSS) are widely employed to mitigate the interference in wireless communication systems [1-2]. In FHSS systems, the modulated symbol is frequency hopped by the different frequency carrier which is pseudorandomly determined. In the receiver, the received signal is frequency-dehopped synchronously. It is difficult for the frequency synthesizers to maintain phase coherence between successive hops over a wide hop bandwidth and it is impossible to perform differential detection within one hop using fast frequency hopping, so FHSS systems use noncoherent M-ary Frequency Shift Keying (MFSK) modulation schemes.

FHSS systems with partial band interference require proper combination of spread spectrum modulation, error correcting code, diversity and decoding method for better performance. An intelligent jammer usually reduces the effectiveness of FHSS, however this effectiveness can be maintained through the use of Error Control (EC) codes. Furthermore, a jammed channel may not be stationary, so channel state information, usually called side information, can be used to improve the performance of EC codes. In this study Reed Solomon (RS) codes are employed in a standard error correction role to provide some Anti-Jamming (AJ) capability. Reed-Solomon error correction is an error-correcting code that works by oversampling a polynomial constructed from the data.

The polynomial is evaluated at several points and these values are sent or recorded. RS code possesses a low probability of decoding error and has an easily implemental algorithm to exploit erasure information.

SYSTEM MODEL

The FHSS system model considered in this study is characterized by partial band jamming. The input binary data is coming at a rate R_b with a symbol rate R_s is given as:

$$R_s = \frac{R_b}{\log_2 M} \qquad (1)$$

Fast Frequency Hopping (FFH) is used so that one of M-ary symbol is transmitted over L independent hops, where L is greater than or equal to one. Perfect side information is assumed available to detect if a hop is jammed. The number L is sometimes called the order or level of diversity. The input and output periods and rate are related, repetitively by

$$L = \frac{R_h}{R_s} \qquad (2)$$

and

$$R_h = L \times R_s \qquad (3)$$

Corresponding Author: Abid Yahya, Collaborative MicroElectronic Design Excellence Center, Universiti Sains Malaysia, Engineering Campus, Nibong Tebal, Penang, Malaysia Tel: +604-593 0801 Fax: +604-593 0920

where, R_h is the hop rate.

The system considered here uses 1MHz separation frequency and 2MHz sampling frequency with a signal length of 5000 bits. The system uses Pseudo Noise (PN) code generator for spreading the bandwidth of the modulated signal to the larger transmission bandwidth and distinguishing between the different user signals utilizing the same transmission bandwidth in a multiple-access scheme[3]. The selected baseband signal is broken into the L hops, by mixing with the output of a synthesizer which controlled by a pseudorandom sequence generator. The synthesizer selects a new frequency every seconds and the output of the mixer is passed through a filter, translated to RF, amplifier and radiated from the transmit antenna.

The model for the partial band noise interference is the same as[4] with an addition of Additive White Gaussian Noise (AWGN) at the receiver. A partial band noise jamming where the jammer transmits noise over a fraction of the total spread spectrum signal band spreads noise of total power P_J evenly over some frequency range of bandwidth W_J, which is a subset of the total spread bandwidth W_{ss}. Fraction (ρ) is defined in[5] as the ratio

$$\rho = \frac{W_J}{W_{ss}} \qquad (4)$$

where, ρ is (0, 1) which is the fraction of the total spread spectrum band that has noise of power spectral density[5]

$$\frac{P_J}{W_J} = \frac{P_J}{W_{SS}} \cdot \frac{W_{ss}}{W_J} = N_J/\rho$$
$$N'_J = N_J/\rho \qquad (5)$$

If N_o denotes that of the AWGN and $\rho(0 \le \rho \ge 1)$ is the probability that a particular dwell time (frequency slot) is jammed, then the one-sided spectral density of the Gaussian noise is $N_o + \frac{N_J}{\rho}$ with probability ρ and it is N_o with the probability $1-\rho$.

There are several issues while considering coding for spread spectrum communications in the presence of partial band interference. One issue is whether or not the decoder knows if the received signal has been jammed or not. Naturally, the decoder knowing and using this side information in order to improve the performance compared to coding without side information available. Interleaving is another issue but the system we are considering in this study is FFH

spread spectrum, then there is one symbol transmitted per hop so interleaving is unnecessary (Fig. 1).
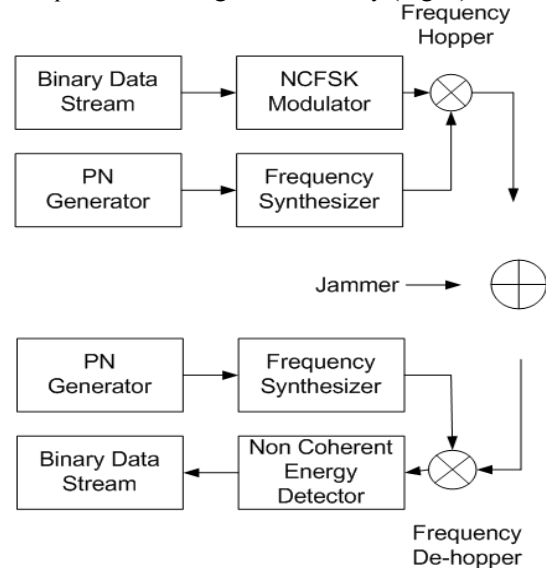

Fig. 1: Structure of the transmitter and receiver of FFH system

PERFORMANCE EVALUATION FOR REED-SOLOMON CODES

Reed Solomon coding can be made more effective by erasing jammed symbols and employing a decoding technique that will correct erasures.In Reed-Solomon code, the encoded data is first visualized as a polynomial. The polynomial is then encoded by its evaluation at various points and these values are what is actually sent. During transmission, some of these values may become corrupted. The data points in RS are sent as encoded blocks. The total number of m-bit symbols in the encoded block is $n = 2^m-1$[6]. The error-correcting ability of any Reed-Solomon code is determined by $e = n-k$, the measure of redundancy in the block. If the locations of the errored symbols are not known in advance, then a Reed-Solomon code can correct up to $t = \frac{n-k}{2}$ erroneous symbols, which is the error capability of the Reed-Solomon (n, k) code where k is the information symbols in a codeword of length n[6]. In this study RS codes over GF (M^m) are employed with m M-ary symbols in each RS symbol. The probability for uncoded system is given by:

$$p_s \le 1 - (1-P_h)^{(K-1)} [(1-\rho)(1-P_o)^m + \rho(1-P_{J,_o})^m] \qquad (6)$$

where, $[(1-\rho)(1-P_o)^m + \rho(1-P_{J,o})^m]$ represents the probability of no error due to Gaussian noise in m M-ary symbols. The probabilities P_o and $P_{J,o}$ represent the error probabilities of an M-ary FSK system with non-coherent demodulation disturbed by AWGN of one sided spectral densities N_o and $N_o + \dfrac{N_j}{\rho}$, respectively.

Where P_h is the probability of a hit from another user and is given by:

$$P_h = \left[1 + \dfrac{m}{N_s}\right]\dfrac{1}{q} \tag{7}$$

where, q is the number of available frequencies.

The symbol error probability for the coded system is given by[7]:

$$P_{e,s} = \sum_{j=t+1}^{n} \dfrac{j}{n}\binom{n}{j} p_s^{\,j}(1-p_s)^{n-j} \tag{8}$$

The probability of an erasure, which can be used by the RS decoder to erase the symbols, is given by:

$$\in_s = \rho + \left[1-(1-P_h)^{(K-1)}\right] - \rho\left[1-(1-P_h)^{(K-1)}\right] \tag{9}$$

The probability of a symbol error is shown in Eq. 10, if the jammer is present and or if interference from other users is present.

$$p_s = \left[1-(1-P_o)^m\right](1-\in_s) \tag{10}$$

where, $(1-\in_s)$ is the probability of no interference from the jammer or from the other users and $[1-(1-P_o)^m]$ is the probability of error due to the thermal noise alone. It maybe noted that neither $\in_s$ or p_s depend on the signal-to-jammer ratio E_b/N_j. The probability of RS symbol error at the decoder is[7]

$$P_{e,s} = \sum_{\substack{j+1\leq n \\ e+1\leq 2l+j}} \dfrac{j+1}{n}\binom{n}{j}\binom{n-j}{j} p_s^{\,j}\in_s^{\,j}(1-p_s-\in_s)^{n-1-j} \tag{11}$$

When the number of erasure is less than or equal to e = n-k, then the probability of error is

$$P_{e,s;1} = \sum_{j=0}^{e}\binom{n}{j}\in_s^{\,j}(1-\in_s)^{n-j}$$
$$\sum_{\substack{e+1\leq 2l+j \\ l+j\leq n}} \dfrac{j+1}{n}\binom{n-j}{j}P_o^{\,l}(1-P_o)^{n-j-1} \tag{12}$$

where, j is the number of erased symbols which are subject to either partial band interference or multiple access interference, whereas n-j is the number of symbols ,which are not erased and l is the number of symbols out of those n-j symbols that resulted in a receiver error due to the thermal noise alone.

When the number of erasure is larger than e = n-k, then the probability of error is given by:

$$P_{e,s;2} = \sum_{j=e+1}^{n}\binom{n}{j}\in_s^{\,j}(1-\in_s)^{n-j}$$
$$\sum_{\substack{t+1\leq l_1+l_2 \\ l_1\leq j \\ l_2\leq n-j}} \dfrac{l_1+l_2}{n}\binom{j}{l_1}\bar{p}^{\,l_1}(1-\bar{p})^{j-l_1}\binom{n-j}{l_2}P_o^{\,l_2}(1-P_o)^{n-j-l_2} \tag{13}$$

In Eq. 13 $\bar{p}$ represents the probability of error when there is partial band or multiple access interference and is given by:

$$\bar{p} = \dfrac{\in_1}{\in_s}\left[1-(1-P_{J,o})^m\right] + \dfrac{\in_2}{\in_s}\left(1-\dfrac{1}{M^m}\right) \tag{14}$$

where, $\in_1$ and $\in_2$ represent the probability of being jammed when not hit by the users and when hit by the users respectively. Which are given by:

$$\in_1 = \rho(1-P_h)^{(K-1)} \tag{14a}$$

and

$$\in_2 = 1-(1-P_h)^{(K-1)} \tag{14b}$$

At the output of RS decoder, the total error of probability is given by:

$$P_{e,s} = P_{e,s;1} + P_{e,s;2} \tag{15}$$

RESULTS AND DISCUSSION

The numbers of frequencies used for frequency hopping are represented by q with noncoherent M-ary FSK modulation. In each of the RS code m the number of M-ary symbols is employed.

One symbol error occurs when 1 bit in a symbol is wrong or when all the bits in a symbol are wrong. In this study popular Reed-Solomon code RS (255, 223) with 8-bit symbols is used which corrects 16 symbol errors. In the worst case, 16 bit errors may occur, each in a separate symbol (byte) so that the decoder corrects

16 bit errors. In the best case, 16 complete byte errors occur so that the decoder corrects 16×8 bit errors. RS
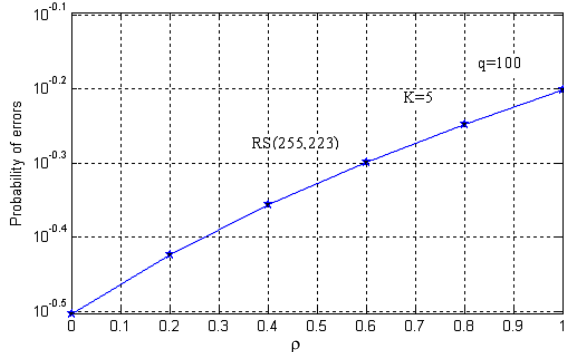

Fig. 2: Probability of errors for coded system using noncoherent 32-ary FSK modulation with q = 100 per 4 words to be processed
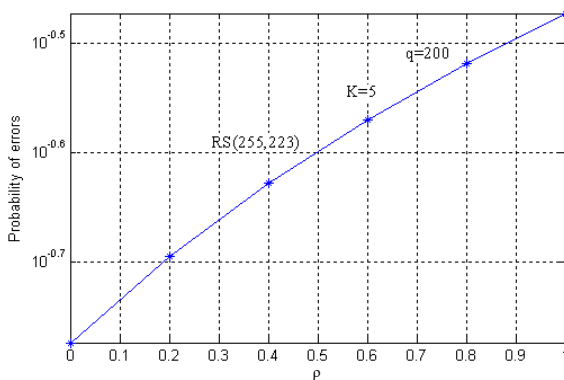

Fig. 3: Probability of errors for coded system using noncoherent 32-ary FSK modulation with q = 200

codes are particularly well suited to correcting burst error where a series of bits in the codeword are received in error.

Noise has been added to RS encoded data, which is demodulated at RS decoder perfectly as shown in Fig. 2 and 3. Matlab simulations have been implemented for numerical results. The probability of an erasure versus ρ is plotted for RS (255, 223) code with 16-bit error correction capability of the code. In order to check the compatibility and performance of the system, using RS as a Forward Error Control coding, we doubled the number of frequencies and the probability of error is slightly increased which is negligible as compared to the rate of frequency as shown in Fig. 2. The simulation results have shown that system, having RS (255,223) code with M = 32 requires

substantially less signal to jammer power ratio to achieve the same bit error probability. Therefore the overall performance of the error erasure correction decoding is better than that with error correction decoding alone. Numerical results for other cases while using the available formulas will be discussed in the forthcoming papers.

CONCLUSION

There are many ways in which hardware savings can be achieved if less performance is required. This study provides an analytical framework for evaluating the performance of coded FHSS systems operating in the presence of partial band noise jamming. The RS coding is used as forward error control coding which treats each codeword of the orthogonal code as a single character in its large alphabet. However, the RS codes have the remarkable property that they able to correct any character erasures. The numerical results have proved the performance of the error erasure correction decoding.

REFERENCES

1. Levitt, B.K. and J.K. Omura, 1981. Coding tradeoffs for improved performance of FH/MFSK systems in partial band noise. In: IEEE Natl. Telecommun. Conf. Rec., IEEE Cat. no. 81CH1679-0, pp: D9.1.1-5.
2. Levitt, B.K., 1982. Use of diversity to improve FH/MFSK performance in worst case partial band noise and multitone jamming. IEEE Milcomm. 82., IEEE Cat. No. 81CH1734-3, pp: 28.2.1-5.
3. Schmaltz, M.D., 1990. Impact of Radio Frequency Interference on S-band Communication Mission Performance. NASA/Johnson Space Center presentations.
4. Simon, M.K., J.K. Omura, R.A. Sxholtz and B.K. Levitt, 1985. Spread Spectrum Communications. Computer Science Press.
5. Don, T., 2005. Principles of Spread Spectrum Communication System. Springer, New York.
6. www.wikipedia.org/wiki/Reed%E2%80%93 Solomon.
7. Berlekamp, E.R., 1980. The technology of error correcting codes. Proc. IEEE., 68: 564-593.