

A Numerical Treatment of Air Flow Model in the Area Under the Station Platform of Thailand BTS Sky Train

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Abstract: Problem statement: The area under Phayathai station platform of sky train in Bangkok, Thailand, has a problem of air pollution control. **Approach:** The Bangkok Mass Transit System Company tries to set up the electric fans inside the area for air flow improvement. **Results:** The flow of the air is still not smooth and the air quality is still lower than standard. The assumption of the research that is the flow obstructs by the platform structures. **Conclusion:** In this research, a mathematical model can be simulating the causes of the flow obstacle. The numerical solution of the model is obtained by using a finite element technique.

Key words: Air flow model, flow velocity, numerical treatment, velocity potential, mathematical model, Bangkok sky train, finite element, governing equation

INTRODUCTION

The mathematical models of the small particles transport by wind are considered to many geophysical (El-Baz *et al.*, 1990) and environmental problems (Sharan *et al.*, 1996). The dynamics of each transport are governed by the wind speed, the sizes of particles and the turbulent eddies that disperse and diffuse the particles. The case includes smoke from industrial chimneys and small snow flakes (Smith, 1957) and (Takeuchi, 1980) to fine soil particles. The wind speed has a logarithmic profile (Pasquill, 1962), and then the turbulence of the air flow is an important factor of the model accuracy (Townsend, 1976). For the moderate wind speeds, (Gillette and Goodwin, 1974) studied the simplest possible form of diffusion equation which arise frequently in the air pollution problem area. Hassan and Eltayeb (1991; 1992), extended to concern the variation of concentration to find that the distribution of the concentration is very much dependent on the distance of the wind direction. Hassan and Eltayeb (1993), they extend their study to the saturation in which the source is at the ground level. The effect of air pollution on ozone layer thickness on troposphere over the state of Kuwait (Al Jeran and Khan, 2009), and the effect of size of heated obstruction within room on three dimensional air flow has been characterized (Ahmed *et al.*, 2010). Pochai (2009), gives the mathematical model to investigate the air flow in the area under the BTS station platform. The governing equation with the boundary condition is given. In this research, the finite element method is used to solve the model.

MATERIALS AND METHODS

The mathematical model:

The governing equation: The two-dimensional flow of the air in the domain Ω in the rectangular ordinate (x, y) is considered. If the flow is irrotational, then the velocity components u and v are express in terms of velocity potential Φ as follows (Ninomiya and Onishi, 1991):

$$u = \frac{\partial \Phi}{\partial x}, v = \frac{\partial \Phi}{\partial y} \quad (1)$$

In this problem, the affect of compression will be neglected. Then the equation of continuity becomes:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2)$$

Substitution Eq. 1 into Eq. 2, we get the equation:

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} = 0 \quad (3)$$

The domain boundary $\Gamma = \partial\Omega$ can be classified into two types: S_1 the velocity potential Φ_B is specified and S_2 the normal component of the velocity V_n is specified and total boundary Ω is $\Omega = S_1 \cup S_2$. The boundary condition on S_1 and S_2 are:

$$\Phi = \Phi_B \quad \text{on } S_1 \quad (4)$$

$$\frac{\partial \Phi}{\partial n} = V_n \text{ on } S_2 \quad (5)$$

Boundary condition: By the Fig. 1, it is obtained that the flow in two-dimensional X, Y planes is uniform speed in X-direction. A column with the circular cross section of radius 1.5 m is placed perpendicularly to the flow field. Since the air flow is assumed to be inviscid and irrotational, it is obtained that the flow around the column is expressible as a potential flow.

By the symmetry of the problem, an upper half of the full geometry will be considered. The part of the left boundary AB represents the inlet, the air flow velocity $u = 1 \text{ m sec}^{-1}$, $v = 0 \text{ m sec}^{-1}$. On boundary BC and DE are lines of symmetry. The semicircle CD is assumed to be a slipping boundary. Then the normal component of the air flow velocity will be zero. The left boundary EF is the outlet of the flow that the air is assumed to flow out at the uniform velocity. Then the tangential component of flow velocity will be zero. The top boundary AF is a far field boundary, which is non affective from the column. Then the boundary AF forms a streamline.

The revision of the boundary condition for the area under the station platform of Bangkok sky train: The revised the boundary conditions in terms of the velocity potential Φ as follows, $\Phi = 0$ along the inlet AB, the velocity to BCDE and AF are equal to zero, $V_n = 0$ in Eq. 4. Since the distance between the front and the end of the platform is 30 m. and using Eq. 6 of (Pochai, 2009), it is obtained that the air flow outlet velocity Φ along boundary EF is:

$$\Phi = \int_0^{30} u dx = 30 \text{ m}^2 \text{ sec}^{-1}$$

It is obtained the summarize of the boundary condition for the mathematical model of the air flow in the area under the station platform of Bangkok sky train for Eq. 3 as Fig. 2.

Finite element technique:

The discretisation of the weak form: Consider the discretisation of the weak form corresponding to Eq. 3 using the finite element method. Let ω be weighting functions, being arbitrary but $\omega = 0$ on S_1 , where the value of Φ is known. The weighted residual form of Eq. 3 is obtained by:

$$\iint_{\Omega} \left(\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} \right) \omega d\Omega = 0 \quad (6)$$

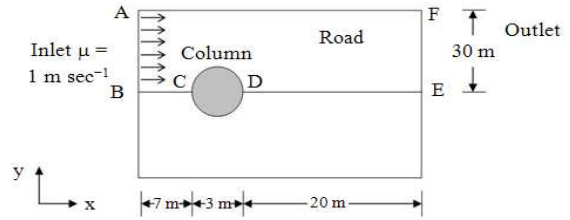


Fig. 1: The area under the station platform of Bangkok sky train with a column (Pochai, 2009)

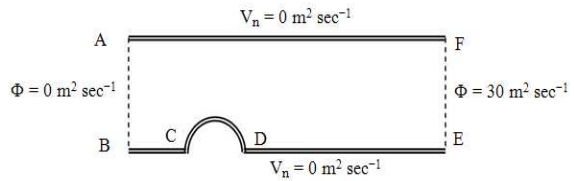


Fig. 2: The revision of boundary condition (Pochai, 2009)

Integration by parts leads to:

$$\iint_{\Omega} \left(\frac{\partial \Phi}{\partial x} \frac{\partial \omega}{\partial x} + \frac{\partial \Phi}{\partial y} \frac{\partial \omega}{\partial y} \right) d\Omega - \int_{S_2} \frac{\partial \Phi}{\partial n} \omega d\Gamma = 0 \quad (7)$$

By substituting the boundary condition of Eq. 5 into Eq. 7 to obtain the weak form:

$$\iint_{\Omega} \left(\frac{\partial \Phi}{\partial x} \frac{\partial \omega}{\partial x} + \frac{\partial \Phi}{\partial y} \frac{\partial \omega}{\partial y} \right) d\Omega = \int_{S_2} V_n \omega d\Gamma \quad (8)$$

The domain is subdivided into triangular element. The following interpolation functions are used for the weighting functions:

$$\Phi(x, y) = \sum_{\alpha=1}^3 T_{\alpha}(x, y) \Phi_{\alpha}, \quad (9)$$

$$\omega(x, y) = \sum_{\alpha=1}^3 T_{\alpha} \omega_{\alpha}, \quad (10)$$

where the shape function T_a are given by:

$$T_{\alpha} = \frac{1}{2\Delta^e} (a_{\alpha} + b_{\alpha}x + c_{\alpha}y) \quad (11)$$

and:

$$\begin{aligned} a_{\alpha} &= x_{\beta}y_{\gamma} - x_{\gamma}y_{\beta}, b_{\alpha} = y_{\beta} - y_{\gamma}, \\ c_{\alpha} &= x_{\gamma} - x_{\beta}, \Delta^e = (b_{\alpha}c_{\beta} - b_{\beta}c_{\alpha}) / 2, \end{aligned} \quad (12)$$

In which $\alpha, \beta, \gamma = 1, 2, 3$. Then the weak form Eq. 8 in each triangular element e is:

$$\sum_{\alpha=1}^3 \left(\iint_e \left(\frac{\partial \Phi}{\partial x} \frac{\partial T_{\alpha}}{\partial x} + \frac{\partial \Phi}{\partial y} \frac{\partial T_{\alpha}}{\partial y} \right) d\Omega \right) \omega_{\alpha} = \sum_{\alpha=1}^3 \left(\int_{S_2^e} V_n T_{\alpha} d\Gamma \right) \omega_{\alpha} \quad (13)$$

where, $S_2^e = S_2 \cap \partial e$. For each triangular element, we obtain the element equations:

$$\sum_{\beta=1}^3 D_{\alpha\beta}^e \Phi_{\beta} = F_{\alpha}^e \text{ for all } \alpha = 1, 2, 3, \quad (14)$$

Where:

$$D_{\alpha\beta}^e = \iint_e \left(\frac{\partial \Phi_{\alpha}}{\partial x} \frac{\partial \Phi_{\beta}}{\partial x} + \frac{\partial \Phi_{\alpha}}{\partial y} \frac{\partial \Phi_{\beta}}{\partial y} \right) d\Omega, \quad (15)$$

$$F_{\alpha}^e = \int_{S_2^e} V_n T_{\alpha} d\Gamma \quad (16)$$

The matrix form of equation: From Eq. 11, it can be obtained the matrix form:

$$[D^e] = \frac{1}{4\Delta^e} \begin{bmatrix} b_1^2 + c_1^2 & b_1 b_2 + c_1 c_2 & b_1 b_3 + c_1 c_3 \\ b_1 b_2 + c_1 c_2 & b_2^2 + c_2^2 & b_2 b_3 + c_2 c_3 \\ b_1 b_3 + c_1 c_3 & b_2 b_3 + c_2 c_3 & b_3^2 + c_3^2 \end{bmatrix} \quad (17)$$

and:

$$\{F^e\} = \frac{1}{2} V_n l \begin{Bmatrix} 0 \\ 1 \\ 1 \end{Bmatrix} \quad (18)$$

In our problem, we assumed that V_n is constant on the boundary segment S_2^e , that length is l . After assembly of all element equations Eq. 14, it can be obtained:

$$[D]\{\Phi\} = \{F\} \quad (19)$$

The finite element solution: Since the nodal values Φ_{α} are known and using Eq.(1), the velocity components u and v is obtained by:

$$u = \sum_{\alpha=1}^3 \frac{\partial T_{\alpha}}{\partial x} \Phi_{\alpha} = \frac{1}{2\Delta^e} \sum_{\alpha=1}^3 b_{\alpha} \Phi_{\alpha} \quad (20)$$

$$v = \sum_{\alpha=1}^3 \frac{\partial T_{\alpha}}{\partial y} \Phi_{\alpha} = \frac{1}{2\Delta^e} \sum_{\alpha=1}^3 c_{\alpha} \Phi_{\alpha} \quad (21)$$

RESULTS

Numerical experiment: We mesh the domain into 180 triangular elements with 115 nodes as show in Fig. 3 and 4. From the revision of the boundary, we have the inlet and out let boundaries along AB, EF and elsewhere are $\Phi = 0$, $\Phi = 30$ and $V_n = 0$, respectively. The calculated results by using the velocity potential are shown in Fig. 5 and Table 1.

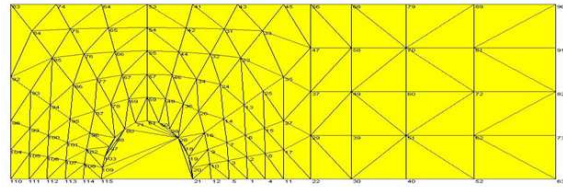


Fig. 3: The domain discretisation

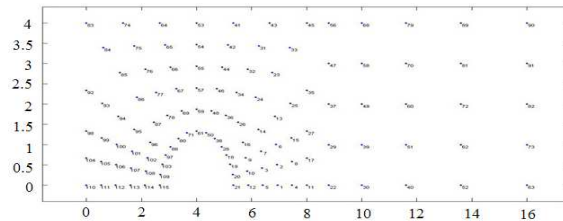


Fig. 4: The nodes numbering

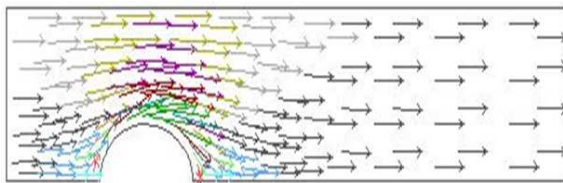


Fig. 5: The flow velocity

Table 1: The calculated flow velocity of some elements of most inside area of the platform

Element number	Velocity in X-direction u (m sec ⁻¹)	Velocity in Y-direction v (m sec ⁻¹)
89	0.7256	-0.1677
81	0.6801	-0.0017
72	0.7583	-0.0009
62	0.7357	0.0391
52	0.6834	0.0511
90	0.7798	-0.3263
91	0.6286	0.0019
82	0.7001	-0.0287
73	0.8080	-0.0014
63	0.7576	0.0659

DISCUSSION

The flow velocity of the most inside area of the platform is slower than most outside area. These means that the columns of platform must be a cause of air flow obstacle.

CONCLUSION

The governing equation with the revised boundary condition for BTS sky train platform is presented. Accordingly, a finite element technique for simulation the air flow and the numerical results are also given. It is obtained that columns of platform is a cause of the air flow obstacle. For the future work, another platform structures could be considered. The field measurement and the numerical result of the air flow in the consideration area should be compared.

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