

Original Research Paper

Dialectic Form Finding of Structurally Integrated Adaptive Structures

¹L. Rhode-Barbarigos, ²V. Charpentier, ²S. Adriaenssens and ³O. Baverel

¹Department of Civil, Architectural and Environmental Engineering, University of Miami, Coral Gables FL, USA

²Form Finding Lab, Department of Civil and Environmental Engineering, Princeton University, Princeton NJ, USA

³Laboratoire Navier/Géométrie Structure Architecture, Ecole des Ponts ParisTech, Paris, France

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Corresponding Author:

L. Rhode-Barbarigos

Department of Civil, Architectural and Environmental Engineering, University of Miami, Coral Gables FL, USA

Email: landolfrb@miami.edu

Abstract: Structural engineering, prompted by advances in mechanics and computing as well as design principles such as sustainability and resilience, is evolving towards adaptive structures. Adaptive structures are structures that use active components to change shape and properties in response to their environment and/or to their users' desires. Form-found structures, such as tensegrity and shell structures, can be designed to accommodate such changes within their structural behavior. Dialectic form finding is an extension of the traditional form-finding process integrating performance-related constraints and criteria in the search of a geometry in static equilibrium. Two examples of dialectic form-found structurally integrated adaptive structures are presented. The first example is a shape-shifting tensegrity-inspired structure, while the second example is a shape-shifting shell structure. Both systems are designed to explore elastic deformations for shape changes reducing actuation requirements and highlighting the potential of the proposed method.

Keywords: Adaptive Structures, Form-Finding, Analysis, Shape-Changing, Tensegrity, Shell

Introduction

The idea of adaptive structures can be found on the first manmade tents and temporal shelters. However, the origin of the concept is attributed to kinetic architecture. Kinetic architecture was first introduced by Zuk and Clark (1970) as "form may change very slowly by evolution, moderately fast by the process of growth and decay and very fast by internal muscular, hydraulic, or pneumatic action". An overview of kinetic architecture can be found in (Fox and Kemp, 2009). Although kinetic architecture has been focusing on adaptive structures mostly for movable and shape/space-changing applications, structural engineering has been identifying them as a potential solution to safety and serviceability design problems (Yao, 1972; Housner *et al.*, 1997). The origin of adaptive structures in structural engineering can be attributed to Yao (1972) who introduced the concept of structural control as an alternative approach to

safety problems. However, nowadays adaptive structures are employed for applications that span beyond safety, as structural control against long return-period phenomena is not widely accepted due to cost and reliability issues (Housner *et al.*, 1997; Domer and Smith, 2005). An extensive review of structural control applications can be found in (Korkmaz, 2012).

Adaptive structures were first defined as structures that include both passive and active load-bearing elements (Soong and Manolis, 1987). Wada and Das (1991), who proposed a generalized framework based on various levels of structural control, defined them as structures that include actuators that allow them to change state or characteristics in a controlled manner. Movable and shape-shifting/deployable structures can thus be classified as adaptive structures that can perform large shape changes (Gantes, 2001; Pellegrino, 2001). A more recent definition was given

by Sobek and Teuffel (2002) describing them as systems with the ability to manipulate their internal force distribution or influence their external loads over time. Elements that change length, such as active struts or active cables, have thus been extensively studied for the design and control of adaptive and active structures (Korkmaz, 2012). Nevertheless, the design of adaptive structures remains a challenging task due to the lack of universal methods for the integration and control of active elements. Moreover, active elements and control are often added to the structure after its design has been completed. Therefore, they are cannot be considered as structurally integrated. There are only few examples where the design of the structure and the control system were carried out simultaneously (Lu and Skelton, 1998; Aldrich *et al.*, 2003; Adriaenssens and Ney, 2007; Thrall *et al.*, 2012a; 2012b).

In this study, dialectic form finding is explored as strategy for the design and analysis of structurally integrated adaptive structures. Through the paradigm of a tensegrity and a shell structure, it is shown that form-found structures can be designed to accommodate adaptation within their structural behavior. They are thus efficient both structurally as well as from an actuation perspective. The remainder of this paper is organized as follows: In section 2, the concept of dialectic form-finding is introduced. In Sections 3 and 4, the approach is applied on a tensegrity structure and a shell structure, respectively. Conclusions and further discussion are presented in section 5.

Dialectic form finding

Form finding traditionally describes the forward process in which parameters, such as topology and support conditions, are controlled to find a geometry which is in static equilibrium with a specific set of design loads (Adriaenssens *et al.*, 2014a). As a result, the obtained structures exhibit material and structural efficiency. Numerical form-finding methods include methods such as the force density (Schek, 1974), thrust network analysis (Block, 2009), dynamic relaxation (Barnes, 1999) and non-linear Finite Element (FE) approaches (Pagitz and Mirats Tur, 2009). Extensive reviews of form-finding methods can be found in (Tibert and Pellegrino, 2003; Veenendaal and Block, 2012; Adriaenssens *et al.*, 2014b). In this study, numerical form-finding methods are combined with shape and structural behavioral constraints to develop adaptive structures that integrate active elements within their structural system. This approach is referred to as dialectic form finding (Adriaenssens *et al.*, 2014b) and is illustrated in Fig. 1.

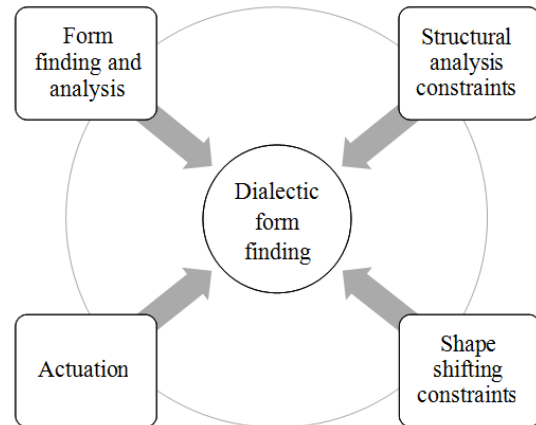


Fig. 1. Illustration of the dialectic form-finding approach and its components

Dialectic form finding describes an iterative process with four components: Form finding and analysis, structural analysis constraints, shape shifting constraints and actuation. The process starts with a statically stable configuration where actuation (element elongation or external load) is applied. A form finding and analysis method is employed to identify a new intermediate equilibrium configuration. The new configuration is then checked for structural performance and shape compliance depending on the system and application in question. If there is a constraint violation, a new configuration can be found by adapting the actuation applied. However, in case that modifying the actuation applied has little or no effect in the constraint violation or does not allow the desired shape transformation, the initial design should be re-examined and modified. If all constraints are satisfied, the process continues with a new actuation and the identification of a new equilibrium configuration. Under the assumption of quasi-static actuation, shape/state transformations are thus based on a series of intermediate form-found stable equilibrium configurations.

A Structurally Integrated Adaptive Tensegrity Structure

Tensegrity structures are defined as spatial reticulated structures composed of elements with unilateral and bilateral rigidity (Motro, 2006). Elements are combined in a self-equilibrated pre-stressed state that provides stability and stiffness to the structure. The tensegrity concept exists for almost 60 years now and has received significant interest from scientists and engineers (Munghan and Abel, 2011; Rhode-Barbarigos *et al.*, 2012a; Ingber *et al.*, 2014; Kim *et al.*, 2014) especially for adaptive/shape-changing applications as actuators

and structural elements can be combined (structurally integrated actuation) (Adam and Smith, 2008; Veuve *et al.*, 2015). Tensegrity structures have been traditionally developed and analyzed using form-finding methods in which bilateral rigidity elements are modeled as truss elements (elements experiencing tension or compression). However, in reality tensegrity elements sustain also some bending. Bending might be negligible compared to the axial forces if it originates from dead load, but it might become critical if imperfections such as initial curvature or eccentricities are present (Rhode-Barbarigos and Adriaenssens, 2014). However, if designed appropriately, tensegrity structures can actually benefit from bending similar to other bending-active systems (Lienhard *et al.*, 2011a).

The tensegrity structure employed in this paradigm is the simplex module (Motto, 2006). The simplex is composed of three struts and nine cables (Fig. 2). Dynamic relaxation is employed to investigate the actuation requirements (number of active elements) for folding and unfolding as well as the structural response of the module throughout shape transformations. Dynamic relaxation is an explicit numerical form-finding and analysis method of tensile structures (Day, 1965; Adriaenssens 2008; Ali *et al.*, 2011; Richardson *et al.*, 2013; Siu *et al.*, 2013; Segal *et al.*, 2015;) that avoids stiffness-matrix calculations (Brew and Brotton 1971) exploring the fact that the static solution for a structure subject to loading is the steady state of a step-force damped vibration. In this study, the spline-element formulation by Adriaenssens and Barnes (2001) is employed in the dynamic relaxation scheme to study elastic deformations due to bending.

In order to investigate the effect of bending action in the module, elements are first modeled using truss elements (purely axially loaded elements) and then using spline elements (initially straight elements that are actively bent). In order to initiate bending numerically in the spline elements, an initial curvature $l/l_{s,e}$, where $l_{s,e}$ is the spline-element length, is given to the elements. Cables are made of stainless steel while trusses and

splines are composed of Carbon Fiber Reinforced Polymer (CFRP). CFRP is chosen for its low Young modulus and high bending strength that allow the exploration of active bending deformations (Lienhard, 2014). The geometrical and material properties of the tensegrity module are given in Table 1.

In this study, cable-length changes are employed to fold and unfold the module. Changing cable lengths allows passing from one equilibrium configuration to another using finite mechanisms, while ensuring stability and stiffness (Rhode-Barbarigos *et al.*, 2012b). The module has a height of 1m and a width of 0.87 m. To fold the simplex to half of its height, the length of horizontal cables has to increase while the length of vertical cables has to decrease. To unfold the module, the length of vertical cables has to increase while the length of horizontal cables has to decrease. Consequently, length changes (and thus actuation) in all nine cables of the module are required to change its shape. Moreover, boundary nodes must allow the required in-plane movement. They are thus assumed blocked only along the vertical axis of the module. Figure 3 shows three side-view snapshots of the shape transformation of the tensegrity module with heights of 0.825, 0.75 and 0.625 m respectively.

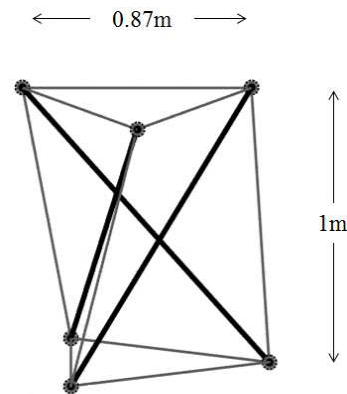


Fig. 2. Illustration of the simplex module with 3 struts (bold lines) and 9 cables (thin lines)

Table 1. Geometrical and material properties of the tensegrity modul

Element	Property	Value
Cable	Length L (m)	0.87 and 1.39
	Tensile strength σ_{x_max} (N/mm ²)	1096.5
	Cross-sectional area A (mm ²)	28.27
	Young's modulus E (N/mm ²)	115'000
Truss/spline	Length L (m)	1.4
	Tensile strength σ_{x_max} (N/mm ²)	2100
	Compression strength σ_{c_max} (N/mm ²)	700
	Bending strength σ_{b_max} (N/mm ²)	2100
	Diameter D (mm)	76.1
	Thickness t (mm)	4
	Young's modulus E (N/mm ²)	40'000

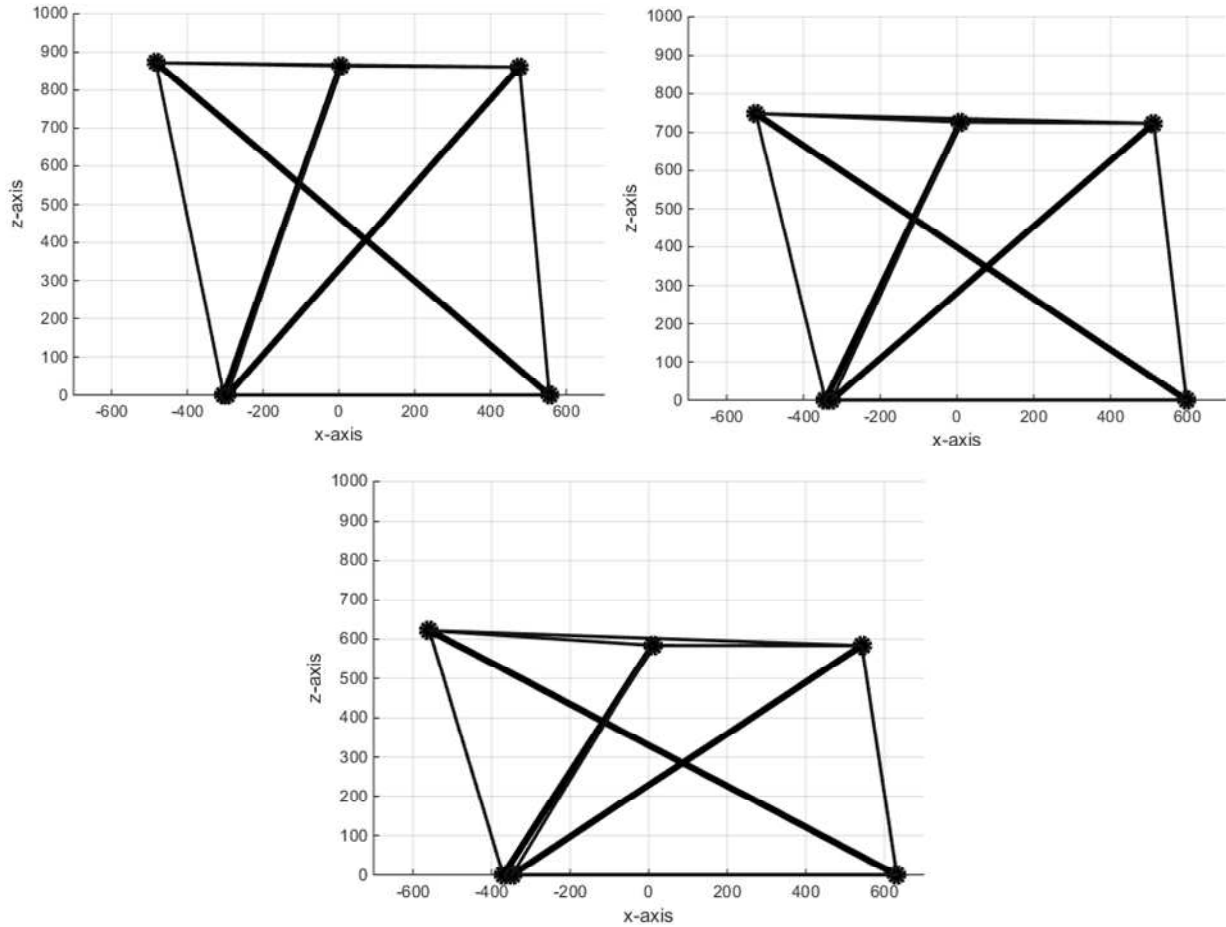


Fig. 3. Side-view snapshots of the shape transformation of the tensegrity module with finite mechanisms assuming strut and cable elements

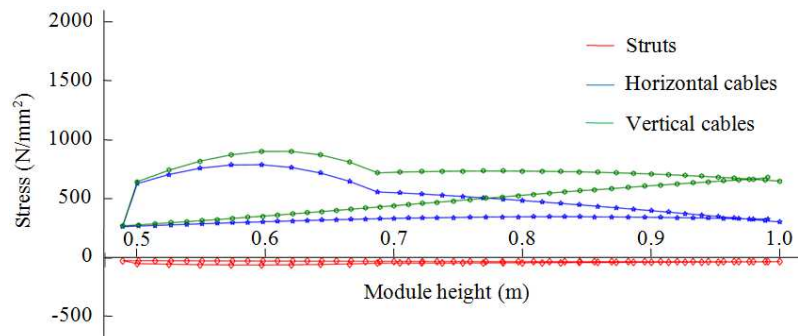


Fig. 4. Evolution of stresses through a folding/unfolding cycle using finite mechanisms assuming truss and cable elements

Figure 4 shows the evolution of stresses in the elements of the module through a folding/unfolding cycle using finite mechanisms and assuming strut as well as cable elements. Since shape transformations are based on a series of finite mechanisms that are controlled (induced and stabilized) by the nine active cables, the stress state in the module can be maintained relatively low compared to design values. Although stresses in the

cables vary during folding and unfolding (vertical cables are the primary folding actuators and horizontal cables are the primary unfolding actuators), stresses in the strut elements remain at approximately the same level.

Bending action can be used as an alternative actuation structurally integrated mechanism for unfolding the structure. Consequently, only the three vertical cables have to be actuated to fold and unfold

the simplex module. Figure 5 shows three snapshots of the shape transformation with module height 0.825, 0.75 and 0.625 m respectively. As the three vertical cables shorten during folding, spline elements bend reducing the height of the module.

Figure 6 shows the evolution of stresses in the elements of the module through a folding/unfolding cycle using bending action and three active cables. Since during folding spline elements are bent, stresses in the elements increase with folding. Moreover, strain energy is stored in the structure (primarily in the spline elements) during folding that is then released during unfolding. Vertical cables control thus the unfolding phase by regulating the strain rate in spline elements. Element stresses remain always below design stresses (Table 1). However, higher actuation forces are required for transforming the shape of the module when spline elements are employed.

Employing an integrated bending-active shape-control reduces actuation requirements. However, it

also affects the structural behavior of the module during shape transformations as well as loading. During service, bending stresses can lead to failure at lower loading levels if estimations are done using traditional form-finding and analysis techniques that model tensegrity structures using axial elements (Rhode-Barbarigos and Adriaenssens, 2014). However, if designed appropriately the structure can benefit from bending through stress stiffening effects.

Table 2 compares the average vertical displacement at top nodes for three intermediate configurations with strut and spline elements under a vertical load of 100N applied at the same nodes. Due to the bending action of the deformed spline elements, displacements under the vertical load applied at the top three nodes decrease significantly in the spline configuration. The overall stiffness of the structure is thus higher compared to the truss configuration revealing a stress stiffening effect.

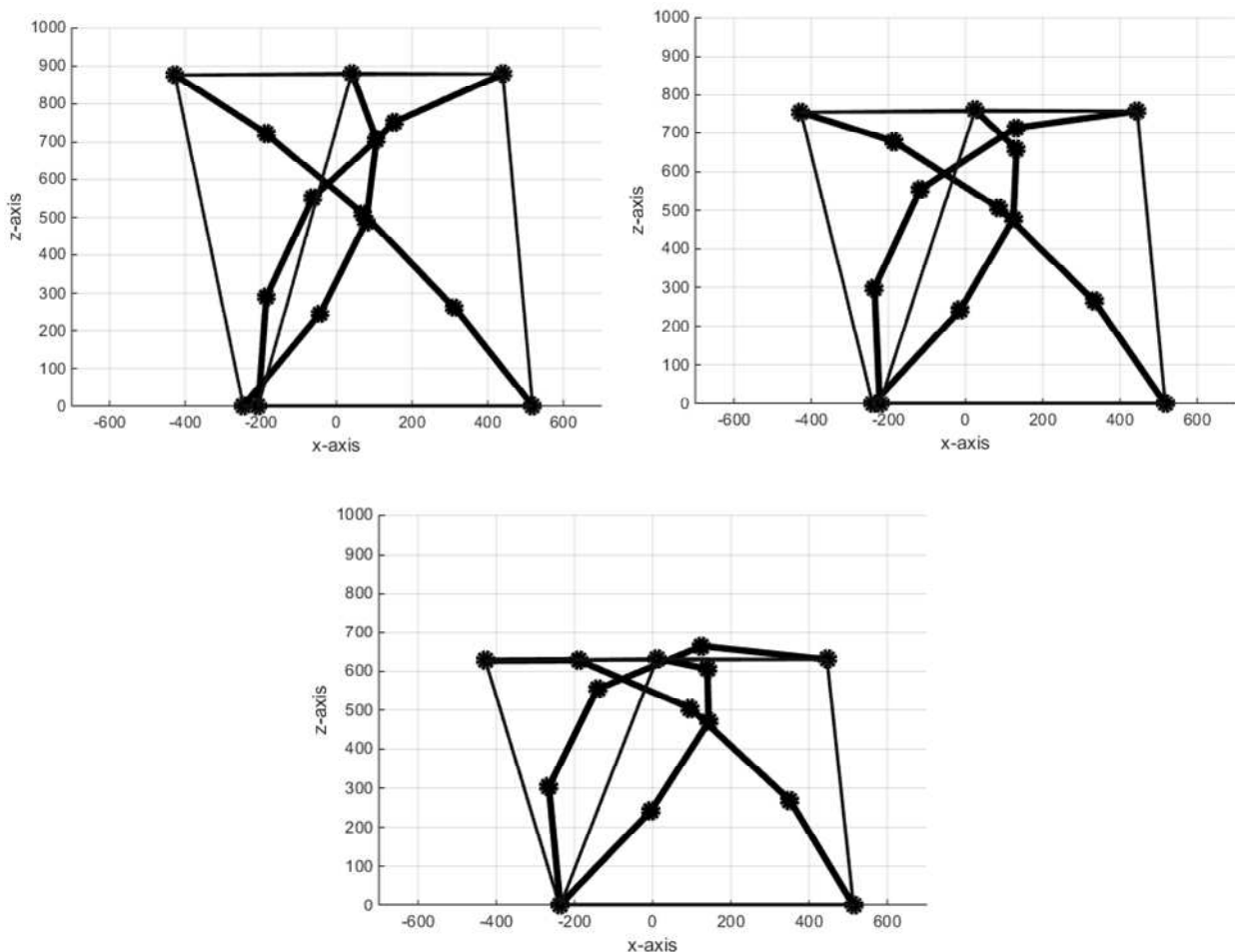


Fig. 5. Side-view snapshots of the shape transformation of the tensegrity module with bending-action assuming spline and cable elements

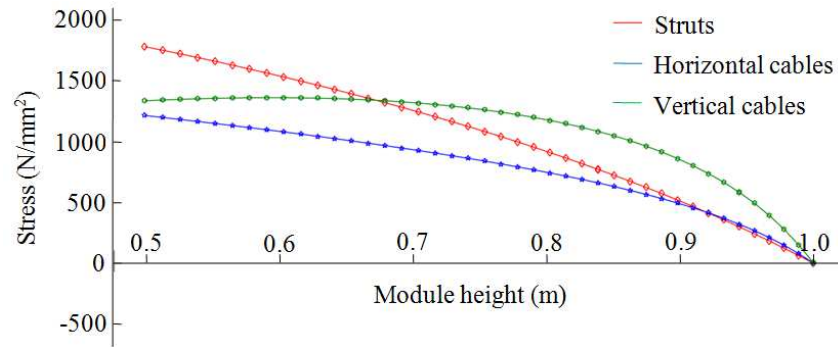


Fig. 6. Evolution of stresses through a folding/unfolding cycle using three active cables and exploring elastic bending deformations in spline elements

Table 2. Average vertical displacement at top nodes for intermediate configurations with truss and spline elements

Module height (m)	Vertical displacement at top nodes (mm)	
	Module with truss elements	Module with spline elements
0.625	7.3	3.7
0.750	10.2	7.5
0.825	14.8	7.4

A Structurally Integrated Adaptive Shell Structure

Shell structures are continuous curved surface structures resisting loading through forces developed in the plane of their surface (membrane stresses). Structural engineers have been investigating and constructing shell structures for decades now (Tedesko, 1937; Nervi, 1955; Candela, 1955; 1963; Isler, 1980). Shell structures offer a great combination of material efficiency and structural performance (Billington, 1982). Thin shell structures have also been studied for their flexibility (Audoly and Pomeau, 2010; Lienhard *et al.*, 2011b), which has been observed in nature (Forterre *et al.*, 2005; Poppinga and Joyeux, 2011) and has great potential for the design of structurally integrated adaptive systems (Lienhard *et al.*, 2011b; Adriaenssens *et al.*, 2014). However, flexible shells involve large displacements and precise kinematics that make the analysis of these systems challenging due to the nonlinearities induced (Calladine, 1989).

The shell structure employed in this paradigm is derived from the shape of the traps of the carnivorous aquatic plant, *Aldrovanda vesiculosa* (Forterre *et al.*, 2005; Poppinga and Joyeux, 2011). The shell structure is composed of two monolithically connected lobes. Actuation in the shell structure occurs from the uniaxial bending of its central part (midrib) that generates a local distributed load which elastically deforms the lobes pushing them inward (closing of the shell). A small elastic deformation in the central part

of the shell results thus in a larger shape change. Moreover, the shell can recover its initial shape once the load has been removed. In the plant structure it is controlled by turgor pressure (Poppinga and Joyeux, 2011), while in the engineered shell structure bending occurs from the application of an external actuation force $F_{closing}$. The hinge-less actuation mechanism of the engineered shell is illustrated in Fig. 7 with the external actuation force $F_{closing}$ represented by a uniformly distributed load directed along the global Z-axis and acting on the surface S_{cd} (highlighted as a bold line).

Numerical simulations using the commercial Finite Element (FE) package Abaqus/CAE (Simulia) are performed to investigate the shape-changing and structural behavior of the shell structure. In this study, the shell structure is initially 0.8 m long and 0.7 m wide (segments AB and EF in Fig. 7, respectively). The shell is simply supported at points A and B. All translations are blocked at point B while only the displacement along the X-axis is free at point A. The loaded surface S_{cd} has a total area of approximately 0.00732 m². The actuation force is a distributed load of 10.93 kN/m on S_{cd} in the Z-direction. To explore elastic deformations, the shell structure is composed of a Carbon Fiber Reinforced Polymer (CFRP). The material properties of CFRP are detailed in Table 3.

Figure 8 shows a series of intermediate configurations during the closing of the shell structure induced by the uniaxial bending of its central part from a gradually increasing force $F_{closing}$. Figure 9 shows the displacement along the Y-axis occurring in the shell

structure (on the two lobes) at each actuation step as well as the corresponding maximal Von Mises stresses. The highest value of Von Mises stresses (291.30 N/mm^2) occurs for the fully loaded/actuated shell (step no.10). However, stresses remain largely below design values as they correspond to 42.9% of the allowable Von Mises stress (678.90 N/mm^2). The kinetic and structural efficiency of the structurally integrated adaptive shell is thus highlighted.

Following the analysis framework proposed in (Lienhard, 2014), the non-linear effect of large displacements induced by actuation on the shell stiffness is numerically determined based on the displacement response of deformed shell configurations to an external loading. The stiffness is valuated as the ratio of applied force over the displacement following Equation 1. The force used to evaluate the stiffness is a uniform surface force of F_{z_test} of 25 N/m applied in the Z-direction towards the interior area of the shell lobes. The stiffness K_z is thus estimated as:

$$K_z = F_{z_test} / U_z \quad (1)$$

where, F_{z_test} is the surface load along the Z-direction on the inside surface of the shell and U_z is the displacement along the Z-axis observed at point F (Fig. 7). Only the Z-axis component of stiffness is considered in this study. For each load step, the surface force F_{z_test} is applied to test the stiffness K_z and extract the deformed geometry. For the extracted geometry at every step, the surface force F_{z_test} is then applied again to test its shape-related stiffness K_c . Figure 10 illustrates the process for the estimation of the shape-related stiffness. In this study, a “stressed” configuration refers to a shell surface that exhibits stresses due to being strained by the actuation loading, while the

“relaxed” configuration corresponds to the extracted geometry of the shell. The relaxed geometry does not exhibit any stresses and strains due to actuation.

Figure 8 shows a series of intermediate configurations during the closing of the shell structure induced by the uniaxial bending of its central part from a gradually increasing force $F_{closing}$. Figure 9 shows the displacement along the Y-axis occurring in the shell structure (on the two lobes) at each actuation step as well as the corresponding maximal Von Mises stresses. The highest value of Von Mises stresses (291.30 N/mm^2) occurs for the fully loaded/actuated shell (step no.10). However, stresses remain largely below design values as they correspond to 42.9% of the allowable Von Mises stress (678.90 N/mm^2). The kinetic and structural efficiency of the structurally integrated adaptive shell is thus highlighted.

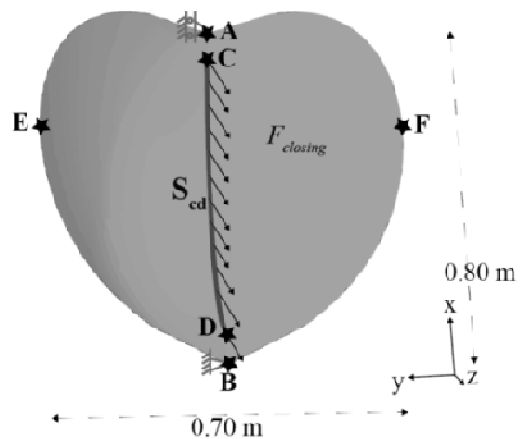


Fig. 7. Illustration of the engineered shell structure and its actuation loading

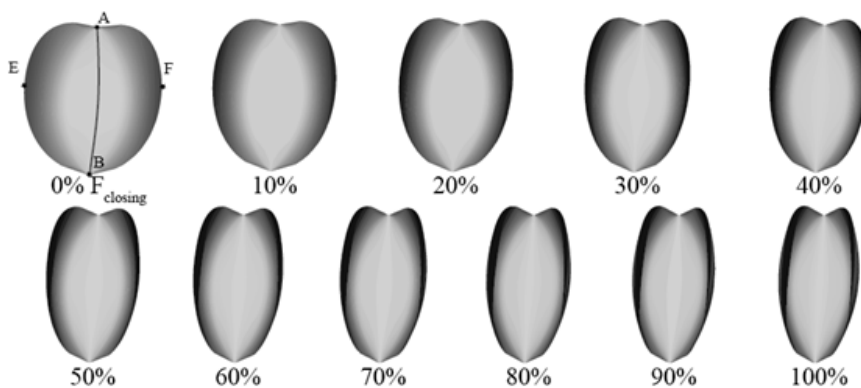


Fig. 8. Series of intermediate form-found equilibrium configurations of the shell structure when actuated by a percentage of the distributed force on its central part

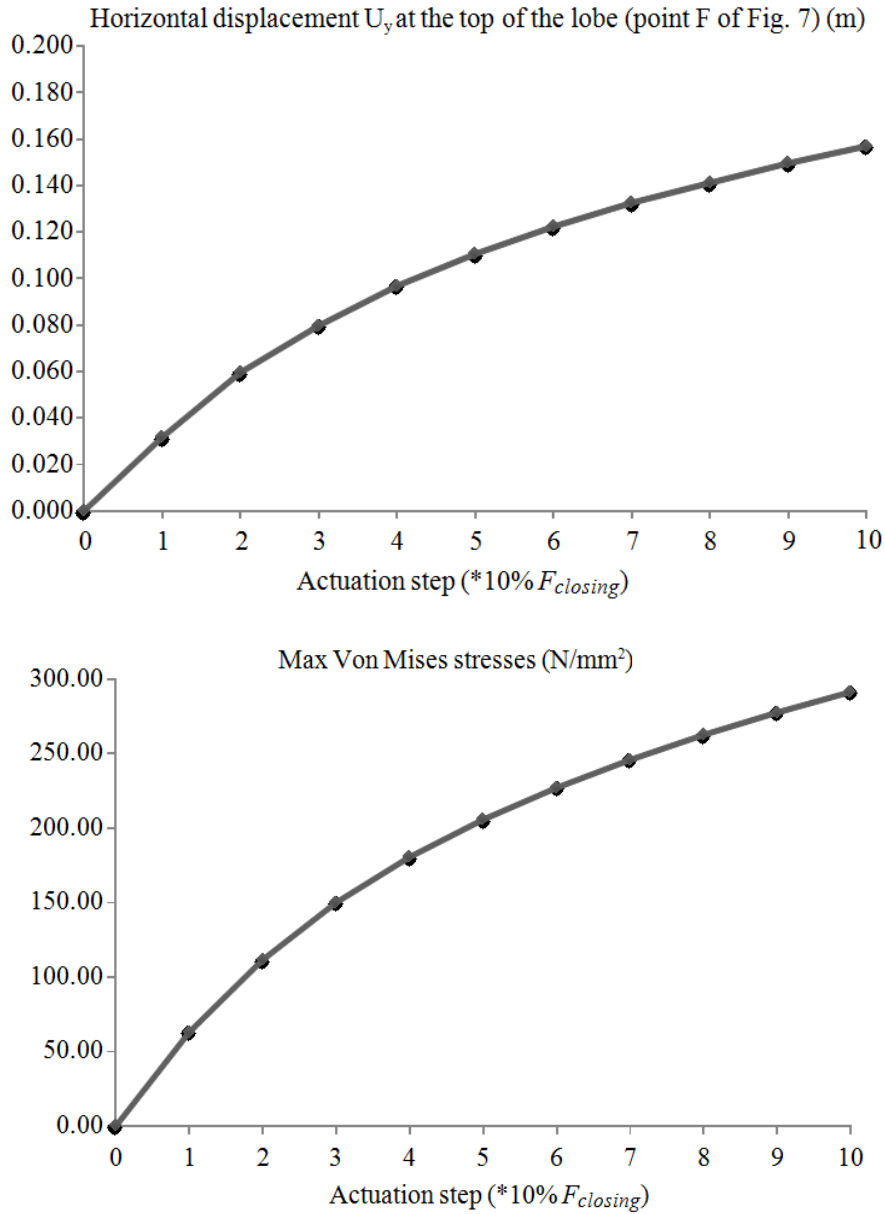


Fig. 9. Horizontal displacements and maximal stresses occurring on the shell structure for each actuation step (*10% of the force $F_{closing}$ on its central part)

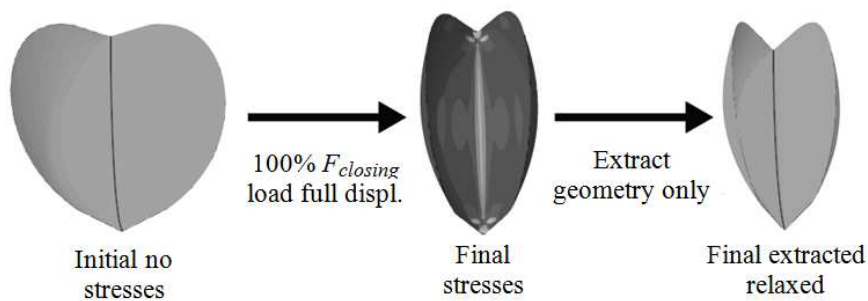


Fig. 10. Methodology employed for the extraction of the geometry after deformation and the estimation of its “relaxed” stiffness

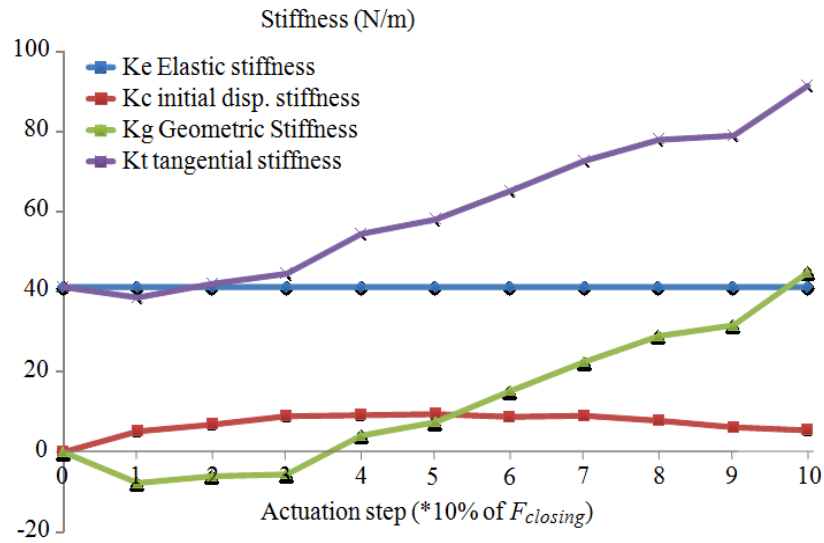


Fig. 11. Progression of the stiffness components of the shell structure during the shape transformation

Table 3. Material properties of Carbon Fiber Reinforced Plastic (CFRP)

Property	Value
Tensile strength	$\sigma_{x_max_tension}$ (N/mm ²) 553
	$\sigma_{y_max_tension}$ (N/mm ²) 297
Compressive strength	$\sigma_{x_max_compression}$ (N/mm ²) 773
	$\sigma_{y_max_compression}$ (N/mm ²) 416
Shear strength	τ_{xy_max} (N/mm ²) 63
Young's Modulus	E_x (N/mm ²) 91'820
	E_y (N/mm ²) 49'550
Poisson's coefficient	ν (-) 0.037
Shear modulus	G (N/mm ²) 4200

In a nonlinear analysis, the tangent matrix K_T is defined as Equation 2 to 6:

$$K_T = K_E + K_C + K_G \quad (2)$$

With:

$$K_E = F_{z_test} / U_{z_initial} = cst \quad (3)$$

$$K_C = F_{z_test} / U_{z_initial} = K_E \quad (4)$$

$$K_G = F_{z_test} / U_z - (K_E + K_C) \quad (5)$$

where, K_E is the elastic stiffness, K_C is the shape-related stiffness (accounts for the change of geometry/curvature of the structure) and K_G is the geometric stiffness. $U_{z_initial}$ is the displacement at points F and E (Fig. 7) for the first load step in the Z-direction and $U_{z_relaxed}$ is the vertical displacement at the same points in the extracted geometry. In the analysis, stiffness components K_C and K_G are determined for each load step. The total load $F_{closing} * S_{CD}$ amounts to 80N and is applied in ten steps of

magnitude $1/10 * F_{closing}$. The constant K_E is given by the response to the first load step. K_C results from the difference of the stiffness of the extracted (stress free) shell with the constant K_E . By applying F_{z_test} to the extracted model, the influence of the stresses developed in the shell up to this load step, is removed from the structural behavior of the model. Finally, K_G is obtained by subtracting K_E and K_C from K_T .

Figure 11 shows the evolution of the tangent stiffness components during the shape transformation of the shell structure. As expected, the elastic stiffness K_E is constant and equal to K_T initially. The shape-related stiffness K_C increases until step 5 and then continuously decreases. The geometric stiffness K_G goes also through two distinct phases: It is negative from steps 1 to 3 and positive from steps 4 to 10. The first (negative) phase of K_G indicates compressive membrane stresses are dominant in the shell structure (Levy and Spillers, 2003), while the second (positive) phase indicates that tensile membrane stresses are governing. Tensile stresses increase the stiffness of the shell structure (tension stiffening) (Levy and Spillers, 2003). During this second phase, K_G increases in average 52% from step to step

until the final step. Overall, K_T is smaller than K_E initially because of the large negative contribution of K_G . However, from step 2 and onwards it becomes larger than KE as tension stiffening occurs. After load step 4, the geometric stiffness K_G controls the behaviour of the tangent stiffness K_T . Consequently, actuation controls the shape of the shell while also affecting strongly its stiffness and consequently its structural response to external loading.

Conclusion

In this study, dialectic form finding, a new approach for the design of adaptive structures is proposed. The proposed approach is an extension of the traditional form-finding process with structural as well as shape related constraints and criteria integrated. The resulting structures incorporate adaptation within their structural behavior. They are thus structurally and adaptively efficient systems. Two dialectic form-finding paradigms were presented. The first one is a tensegrity simplex structure. The study of the tensegrity module revealed that the number of actuated elements can be significantly decreased compared to the conventional finite mechanism shape-shifting control, if elastic bending deformations are explored. However, employing active-bending elements affects also the structural behavior of the tensegrity structure increasing stresses in elements and the overall stiffness of the structure. In the second example, dialectic form finding was applied to a biomimetic shell structure that opens and closes based on the application of an external actuation load. The study revealed that the efficiency of the hinge-less actuation mechanism and the influence of the external actuation load on the stiffness of the structure. As the actuation loading increase, the geometric stiffness of the structure further augments increasing the overall stiffness of the shell structure. Both examples reveal that actuation and structural performance in dialectic form-found structures are closely related. Therefore, if appropriately designed, the resulting adaptive structures can be both structurally as well as adaptively efficient.

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Author's Contributions

Landolf Rhode-Barbarigos and Sigrid Adriaenssens co-developed the research plan. Landolf Rhode-Barbarigos performed also the study on the tensegrity structures and wrote the paper. Victor Charpentier

carried out the FE analysis of the adaptive shell structure. Sigrid Adriaenssens and Olivier Baverel co-advised the analysis of the adaptive shell modules. All authors proof read the paper.

Ethics

The results of the FE analysis of the adaptive shell structure are also part of a conference paper that will be presented at the IASS2015 Annual International Symposium, in Amsterdam, Holland.

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