

## A Preamble-Based Frequency Offset Compensation Scheme in Multiple Input Multiple Output Orthogonal Frequency Division Multiplexing Systems

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**Abstract: Problem statement:** Combining the properties of Multiple Input Multiple Output (MIMO) systems with those of Orthogonal Frequency Division Multiplexing (OFDM), MIMO-OFDM was considered to be a promising technique in the future of wireless communications. However, its sensitivity to frequency offsets which results in Inter-Carrier Interferences (ICIs) makes it necessary to use an exact frequency offset estimation method for data recovery in the MIMO-OFDM receiver. **Approach:** In this study, a new preamble-based frequency offset compensation method was introduced in frequency domain. In each block, two preambles are used to initially obtain the channel coefficients through LS channel estimation method. A polynomial curve fitting algorithm was then applied so that the frequency offsets experienced by every single data subcarrier are separately determined. Finally, to improve the compensating process, an iterative algorithm was applied. **Results:** Simulation results clearly showed that our proposed method was accurate in multipath fading channels and precisely recovered the transmitted data symbols. The BER performance of the iterative algorithm is within an acceptable distance of that in the ideal channel. **Conclusion:** In comparison with the conventional methods, our proposed scheme was less complex and showed better performance in lower SNRs. For further research, it can be investigated and improved while considering correlated antennas and spatial multiplexing.

**Key words:** MIMO-OFDM, frequency offset, estimation, compensation, preamble

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### INTRODUCTION

Orthogonal frequency division multiplexing is being very much interested, in comparison with single carrier techniques, as it divides a high data rate stream into a number of low data rate sub-streams, being simultaneously transmitted on orthogonal subcarriers. As each sub-stream experiences a narrowband flat fading channel, this modulation technique is a proper one to be used in multipath fading channels. As a result of the subcarriers' orthogonality, high spectral efficiency and ICI cancellation are provided. Inter-symbol interferences can also be avoided by using a copy of the last  $N_g$  samples of each block at the beginning of it. This is called Cyclic Prefix (CP) that must be at least equal to the channel length (Stuber *et al.*, 2004).

Using multiple antennas both at the receiver and transmitter, Multiple Input Multiple Output systems are very famous for the high capacities and degrees of freedom that they provide in rich scattering environments (He, 2008).

OFDM combined with MIMO, results in an efficient technique in wireless communications, i.e., MIMO-OFDM. It is necessary to consider a precise frequency offset compensation scheme in a MIMO-OFDM receiver, as it is very sensitive to frequency offsets (IEEE802.11a, 1999) usually caused by doppler shifts or unmatched oscillators.

The receiver uses some known symbols sent by the transmitter to estimate the frequency offsets and to perform the frequency synchronization. Some frequency offset estimation methods have been introduced for MIMO-OFDM systems in recent years. In (Dai and Zhang, 2004), pilots were used for estimating frequency offsets of MIMO-OFDM systems in frequency selective fading channels. In (Jeong *et al.*, 2001), the least square criterion was employed for the frequency estimation. In (Van and Schenk, 2004), training sequences of repeated data were used for synchronizing both fine and coarse frequency offsets in time domain. Finally, in (Mody and Stuber, 2002), repeated data were again used in the training sequences for estimating and compensating coarse and fine

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frequency offsets, sequentially, in frequency and time domains.

In this study, a preamble-based frequency offset compensation scheme is completely introduced. It uses two preambles in each block of data to compensate for all the frequency offsets that it experiences while passing through the channel. Simulation results confirm the good performance of our proposed method in multipath fading channels.

The study is organized as follows: In Materials and Methods, the behavior of a MIMO-OFDM system in presence of the frequency offset is briefly described and our proposed compensation method is completely introduced. In Results the simulation results are discussed. Finally, the study ends with a conclusion in Discussion.

### MATERIALS AND METHODS

**System model:** A MIMO-OFDM system with  $N_t = 2$  transmitter antennas,  $N_r = 2$  receiver antennas and  $k$  subcarriers signaling through an  $L$ -tap frequency selective fading channel is shown in Fig. 1. In presence of frequency offset, the received signal in a MIMO-OFDM system can be described as (Salari *et al.*, 2008):

$$y_i[p] = E(\epsilon) W_k^1 X[p] W h_i[p] + n_i[p] \quad (1)$$

$i = 1, \dots, N_r \quad , \quad p = 1, \dots, P$

in which  $p$  represents the symbol number,  $n_i[p]$  is additive white Gaussian noise with zero mean complex random variables,  $y_i[p]$  is the received multi carrier signal vector defined as:

$$y_i[p] = [y_i[p,0], \dots, y_i[p, k-1]]_{k \times 1}^T ; k = 1, \dots, Nfft \quad (2)$$

Where:

$Nfft$  = The size of fast Fourier transform

$E(\epsilon)$  = A matrix which introduces the frequency offset ( $\epsilon$ ) as:

$$E(\epsilon) = \text{diag}(1, e^{j2\pi\epsilon/k}, \dots, e^{j2\pi\epsilon(k-1)/k})_{k \times k} \quad (3)$$

$W_k^1$  is a matrix which represents the inverse fast Fourier transform performed in the transmitter:

$$[W_k^1]_{r,s} = \frac{1}{\sqrt{k}} e^{j2\pi rs/k} ; r, s = 0, \dots, k-1 \quad (4)$$

$X[p]$  is the transmitted data matrix:

$$X[p] = [X_1(p), X_2(p), \dots, X_{N_t}(p)]_{k \times N_t, k} \quad (5)$$

$$X_j[p] = \text{diag}[X_j(p,0), \dots, X_j(p, k-1)]_{k \times k}$$

$W$  is the fast Fourier transform matrix performed in the receiver side:

$$W = \text{diag}[w, \dots, w]_{N_r, k \times N_t, L} \quad (6)$$

$$[w]_{r,s} = e^{-j2\pi rs/k} ; r = 0, \dots, k-1, s = 0, \dots, L-1$$

and finally,  $h_i[p]$  is the channel vector:

$$h_i[p] = [h_{i,1}^T[p], \dots, h_{i,N_t}^T[p]]_{N_t, L \times 1}^T \quad (7)$$

$$h_{i,j}[p] = [h_{i,j}(p,0), \dots, h_{i,j}(p, L-1)]_{L \times 1}^T$$

$i = 1, \dots, N_r \quad , \quad j = 1, \dots, N_t$

### The proposed frequency offset compensation method:

As described in system model, it is common in literature to consider one frequency offset between transmitter and receiver but in fact it varies due to different arrival angles in the receiver and Doppler effects (Tang and Heath, 2004).

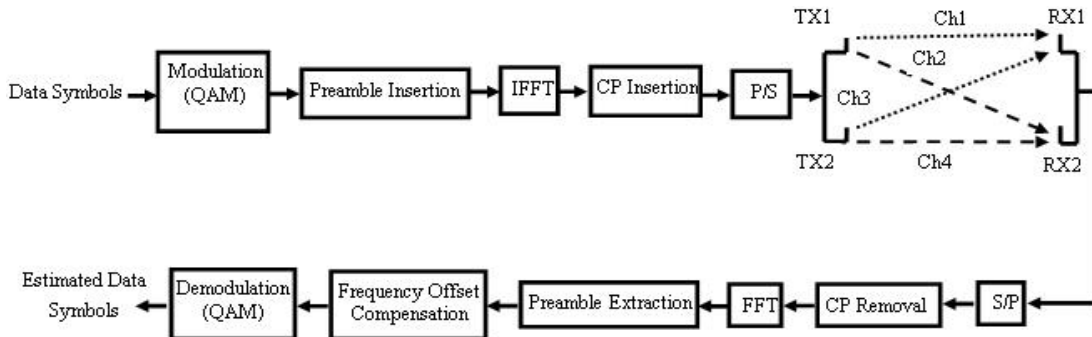


Fig. 1: The structure of a 2x2 MIMO-OFDM system

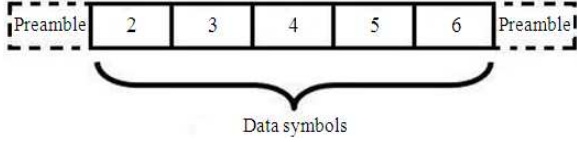


Fig. 2: The block structure

The block structure used in our proposed method is shown in Fig. 2. The first and last symbols of the block are preambles which are the same. The frequency offset is obtained by calculating the angular rotation of every single subcarrier of these preambles, using channel coefficients.

Channel coefficients can be easily calculated using the Least Square (LS) equation as:

$$H(k,p) = (X^H(k,p).X(k,p))^{-1} (X^H(k,p).Y(k,p)) \quad (8)$$

$X(k,p)$  and  $Y(k,p)$  are sequentially the transmitted and received samples on the  $k^{\text{th}}$  subcarrier of the  $p^{\text{th}}$  symbol of the block. As preambles are the first and the last symbols of the block,  $p = 1, 7$  should be used in (8). The frequency offset for each subcarrier of the preambles can now be obtained by:

$$\Delta f_1(k,p) = \frac{\text{angle}(H(k,p))}{2\pi k}; \quad p = 1,7 \quad (9)$$

The subscript 1 is used to represent the first iteration because an iterative algorithm will be later applied to the system. These frequency offsets are then indirectly used for calculating and compensating those of other symbols of the block, i.e. the data symbols.

To do this, a polynomial is formed for each preamble symbol of the block using its relating  $\Delta f_1$  vector and the normalized subcarrier indexes so that:

$$\Delta f_1(k,p) = C_{k,p}(1) \left[ \frac{k}{N_{\text{fft}}} \right]^m + \dots + C_{k,p}(m) \left[ \frac{k}{N_{\text{fft}}} \right] + C_{k,p}(m+1); \quad p = 1,7 \quad (10)$$

The polynomial coefficients of other symbols of the block are obtained by linear interpolation between peer coefficients of these two polynomials as:

$$\hat{C}_{k,p}(j) = C_{k,1}(j) + (p-1) \times \frac{C_{k,7}(j) - C_{k,1}(j)}{6}; \quad (11)$$

$j = 1, \dots, m+1, p = 2, 3, \dots, 6$

The  $\Delta f_1$  vector of each data symbol can now be calculated by substituting the normalized indexes of subcarriers in its relating polynomials:

$$\hat{\Delta f}_1(k,p) = \sum_{j=1}^{m+1} \hat{C}_{k,p}(j) \left[ \frac{k}{N_{\text{fft}}} \right]^{(m+1)-j}; \quad p = 2, \dots, 6 \quad (12)$$

Finally, data symbols are recovered as:

$$\hat{X}(k,p) = Y(k,p) e^{-j2\pi k \hat{\Delta f}_1(k,p)} \quad (13)$$

in which  $Y(k,p)$  is the received signal on the  $k^{\text{th}}$  subcarrier of the  $p^{\text{th}}$  symbol. As  $N_t$  data streams reach to each receiver antenna, the sent data is obtained by:

$$S_i = \text{demodulation} \left( \frac{1}{N_t} \sum_{n=1}^{N_t} \hat{X}_n \right); \quad i = 1, \dots, N_t \quad (14)$$

To more precisely compensate for all the frequency offsets, an iterative process is exerted to the system, i.e., the estimated data are fed back to the system. The whole compensation process is depicted in Fig. 3. This iterative process continues until the required Mean Square Error (MSE) is achieved.

It is important to notice that before estimating the new frequency offsets of each iteration, all the frequency offsets obtained previously are compensated in the received signal:

$$z_n(k,i) = \frac{\text{angle} \left( \left( \hat{X}_n(k,i) \right)^{-1} Y_n(k,i) e^{-j2\pi k \Delta f_{n-1}(k,i)} \right)}{2\pi k} \quad (15)$$

$$\Delta f_n(k,i) = \Delta f_{n-1}(k,i) + z_n(k,i)$$

in which  $n$  shows the iteration number.

The proposed method is a strong one for following channel changes, because the LS estimation is used in the beginning of each iteration. Of course, since two symbols in each block are used for compensating the frequency offsets of other symbols of the block, this method is proper in slowly time variant frequency selective fading channels. It is also important to consider different transmit-receive antenna pairs to be uncorrelated.

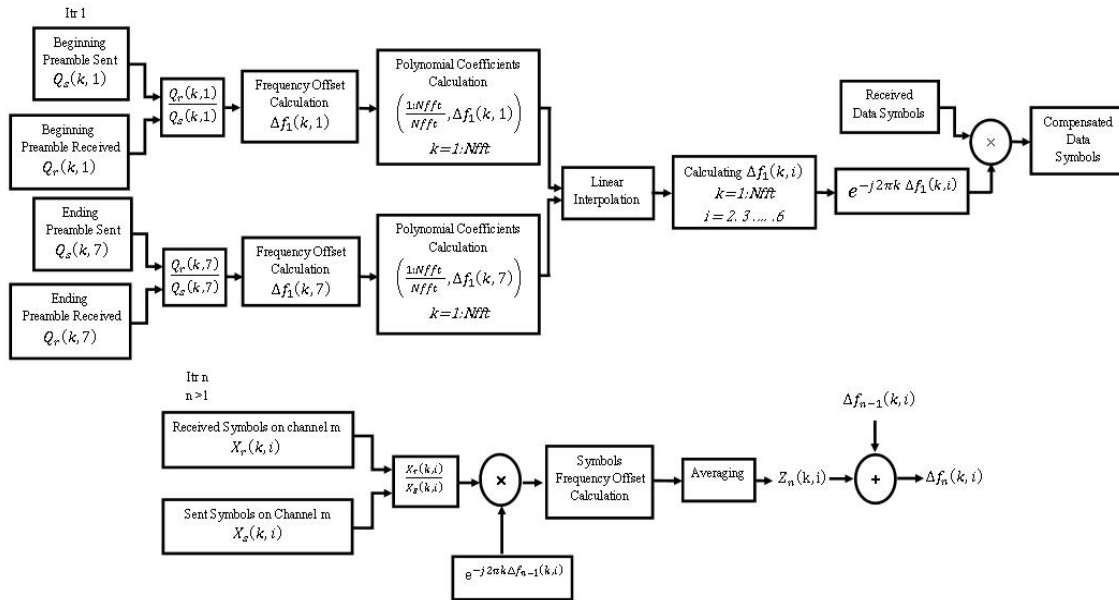


Fig. 3: The proposed frequency offset compensation method

**RESULTS**

A 2x2 MIMO-OFDM system is simulated in matlab software. As mentioned in our proposed method, the first and last symbols of each block are preambles which samples are produced by BPSK modulation. Others are data symbols each of which contains 256 samples chosen randomly from QAM constellation. An FFT size of 256 and a cyclic prefix of 64 are considered in the simulations.

A Rayleigh fading channel with one direct and two indirect links is supposed between transmitter and receiver. Its parameters such as Doppler frequency, delay spread and tap power are chosen, using Stanford University Interim (SUI) channel model (Erceg *et al.*, 2001).

All steps performed in the transmitter are done reversely in the receiver (Fig. 1). After performing FFT, preambles are extracted and channel coefficients are obtained and used for frequency offset compensation as described previously.

An important parameter in determining the precision of our proposed algorithm is the polynomial degree used for initial estimation in the first iteration. Figure 4 depicts the results of a simulation run in order to choose a proper polynomial degree. This 3D graph helps us to trade off between MSE, polynomial degree and Signal to Noise Ratio (SNR). For ease of determination a side view of this graph and its corresponding Bit Error Rate (BER) diagram are sequentially shown in Fig. 5 and 6.

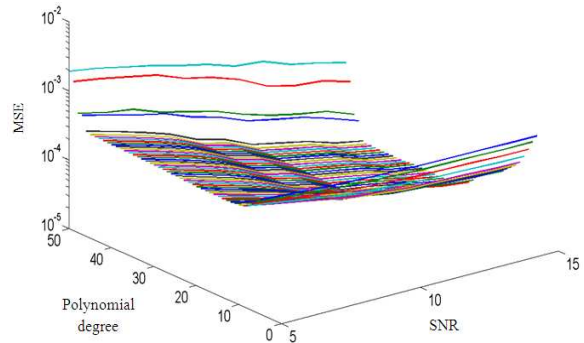


Fig. 4: Trade off between parameters-first iteration

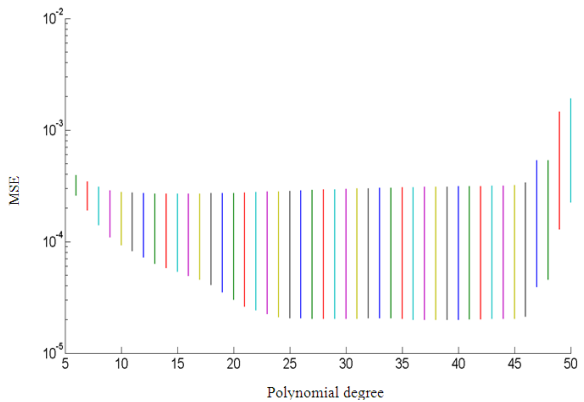


Fig. 5: Side view of Fig. 4

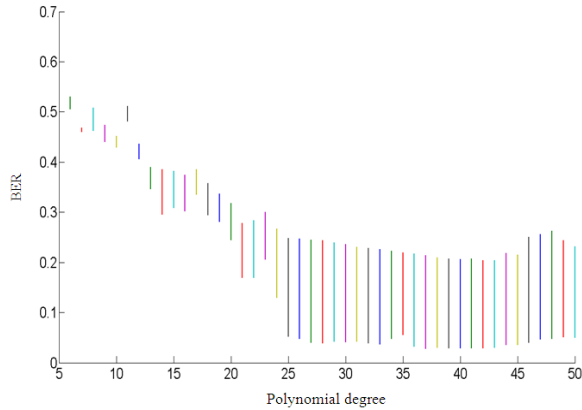


Fig. 6: BER graph relating to Fig. 5

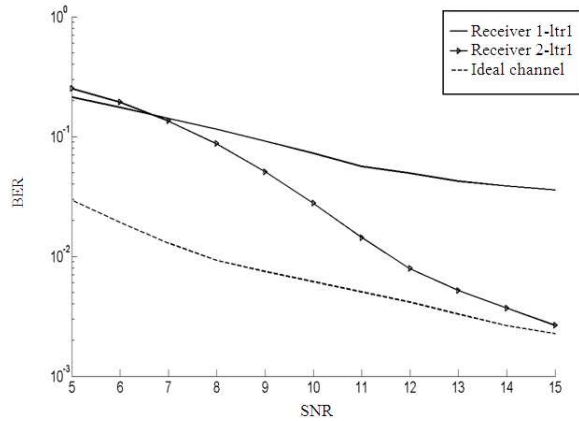


Fig. 7: BER comparison between two receiver antennas-first iteration

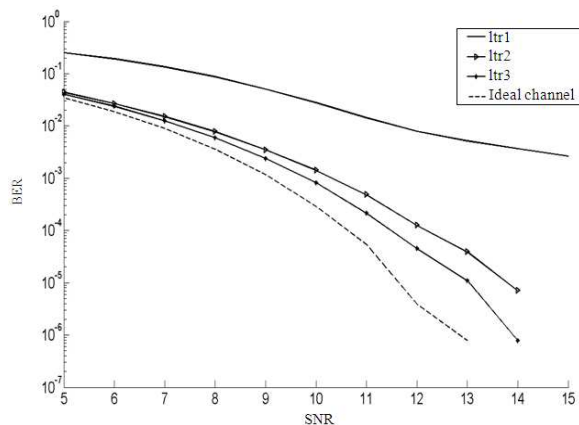


Fig. 8: Applying the iteration algorithm to the system

Comparing Fig. 5 and 6, it is obvious that a polynomial degree of about 40-45 provides us with the highest possible precision.

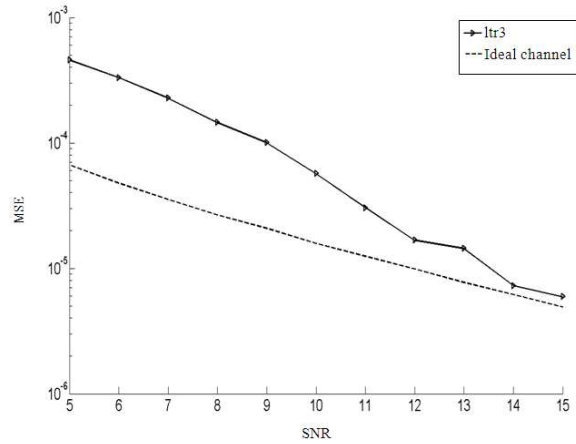


Fig. 9: MSE in different SNRs-third iteration

Considering a polynomial degree of 40, Fig. 7 depicts the BERs obtained for different SNRs in the first iteration of the proposed scheme. Different link parameters such as power and delay result in different performances of the two receiver antennas. As being observed, the second antenna performs more precisely in higher SNRs. So, its output streams are more reliable to be considered for further processing in the following iterations.

Figure 8 shows the improving behavior of the system after applying feed back to it, i.e., in higher iterations the system performance gets near to its performance in ideal channel. But as the number of iterations increases, the speed towards the ideal channel behavior decreases. Therefore, MSE can additionally be used as another criterion for better determining the required number of iterations.

Figure 9 demonstrates the MSEs achieved for different SNRs in the third iteration. It again clarifies that in higher SNRs the system performance gets better and becomes more similar to the performance of the system in the ideal channel. This means a reduction in the frequency offset which results in decreasing of the BER as illustrated in Fig. 8.

## DISCUSION

Effective properties of MIMO-OFDM systems, such as spectral efficiency, resistance to inter-symbol and inter-carrier interferences and good performance in fading channels, encouraged us to investigate it in this study. Channel coefficients were obtained through LS channel estimation method and used separately for determining and compensating the frequency offsets experienced by every single data subcarrier using the polynomial curve fitting method. To improve the

compensating process, an iterative algorithm was also applied to the system which resulted in decreasing of the BER. MSE was mentioned as another important parameter for precisely choosing the proper polynomial degree and the required iterations number. For further research, the proposed method can be investigated and improved while considering correlated antennas and spatial multiplexing.

### CONCLUSION

In comparison with the conventional methods, our proposed scheme was less complex and showed better performance in lower SNRs. For further research, it can be investigated and improved while considering correlated antennas and spatial multiplexing.

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