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Deciding Whether There is Statistical Independence or Not?

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Abstract: The study investigated how students assess statistical independence (SI). A sample of 98 students was available to the researcher. One group was probed on their understanding of statistical independence, a second group was given a proficiency test and a third group was presented a proof for statistical Independence. Interview data from the first two groups exposed student misconception on their understanding and assessment of SI. Those who judge the relation from a causal framework, tend to explore the "representativeness of the relation," if the relation is familiar, students judge it as being dependent; and if the relation is incidental, it is judged as being independent. The results indicate that student proficiency in the calculation and conceptual formalization of the joint probability and conditional probabilities are a prerequisite to the formalization of SI. The study proposes a continuous feedback to student responses through a conceptual mediated approach and use of formal algebra as a viable method to alter strongly held misconception of SI assessment.

Key words: Statsical Independence, Misconceptions in probability and statistics, causality and relations

INTRODUCTION

It is suggested that conditional probability and statistical independence (SI) are difficult concepts in probability theory. It is also suggested that SI is significant because it extends to many statistical methods allowing the modeling of different phenomena[1]. It is found that students of mathematics face considerable difficulty with the assessment of SI that impedes their understanding of more advanced concepts in statistics[2].

Statistical independence is viewed as a fundamental concept in probability and learned in the first courses of statistics at universities[3]. These courses include a short unit on probability where SI is covered in a brief and speedy manner. It is observed that teachers quickly cover statistical and probability concepts with little effort being made in the assessment of student understandings. These conditions provide some opportunities for researchers and educators alike, to devise new teaching methods, to improve teaching through research on student statistics and probability misconceptions.

This study explores difficulties students have in making decisions whether two events are statistically independent or not. The first part of the study explores definitions of statistical independence; second, the study imparts a tutorial interview to conceptually mediate SI through rules of algebra and probability. First, the paper presents an epistemological perspective of the notion of independence and a review of the pertinent literature.

Why Do Students Think Of Independence From A Causal Framework?: It is seldom clear that a distinction is made between the mathematical conception of independence and cause-effect relations[1,4,5,6]. The lay understanding of relations between events draws inferences about cause-effect links. Although, SI is established only by the probabilities, it remains to be an obstacle for naïve learners[7]. Students of statistics may think of SI as part of their understanding of event sequence, that any event (cause) has a consequential effect. They appeal to the connection of antecedence and consequence as an associated relationship and common sense and data logical instances, inconsistent with formal rules[8]. For example, "when ever the maple leaves change their color in Canada, the geese fly South." The change of color and migration of geese are correlated but do not imply causality. Many events in the real world are correlated but believed to imply causation.

The main assumption of this study is that people are more attentive to the psychological impact of causal data as opposed to other forms of informative data. In addition, it is observed that the meaning of words in instruction confuses students for what is meant by dependence or independence. For instance, students consider independent events, having little common elements and judge them as statistically independent. At the heart of the matter, deciding whether events are independent, is inherently different than procedural, or perceptual thinking[9]. SI demands an alternative way of thinking about processing information that combines both, probabilistic and deterministic processes, and appeal less and less to the empirical world. Probabilistic decision-making is analogous to higher level of operational thinking and requires a certain level of abstraction[5, 10].

The Nature Of Causality: In the framework of scientific epistemology, independence of events implies regularity and temporal priority[11]. However, in probability theory, statistical independence is formulated in relation to a number of *n* events. People hold certain beliefs about the nature of things; they manifest a theory of knowledge (epistemology) of how things work in the world. Pertinently, the view that world is seen attached by a web of interrelated entities, communicating intimately together in a chain of causes and effects, suggests that students coming across the notion that nothing is completely independent of anything else, is unusual and not a "regular thing"[12]. More so, the way people make relations between events or express themselves within social interactions mediates causality. In Western thought, scientific reasoning negates the ontic equality of past-present-future conception, that each time frame has its own properties and dimensions and lays the fundamental point and stage for which relations can be formulated through a genre of a chain of connected events, which are made up by a sequences and consequences. Expository text is mediated through a cause and effect link that inform, describe, or explain interrelated ideas. This notion of language, formal and non-formal as it is structured, allows the reader or listener to make appropriate and relevant connections, relationships, for which decisions among and between ideas, concepts, and facts, organize knowledge in an antecedent consequent chain[13]. The outcome is a generated cognitive learning chain of causes and effects representation that imprint a form of thinking about issues, ideas, concepts, and problems framed in an antecedent-consequent scheme. Independence-dependence connection becomes part of a discourse; and a way of thought that uses deeply anchored notions of a relationship[6, 14].

Secondly, use of perceptual faculties could also decide whether a relation is independent or dependent. Namely, a joint event is seen as a dependent relation, such that a joint occurrence provide a mental picture i.e., perceptual representation and an evidence of connection in which one event is dependent on another. As stated by Jenkins and Ward[15], "a single joint occurrence may in some case lead to the conviction that the events are causally related." (p. 1) If dependence is "imagined" as a joint set of common overlaps, students have the tendency to alternate from a causal explanation to a perceptual one in order to decide whether the events are independent.

Literature Tackling Si Through The Use Of N X K Classification Table Structure: SI research was established by Inhelder and Piaget[10] almost six decades ago. They studied children conception of independence through the classification table structure. More recently, Batanero, Estepa, Godino and Green[1] used 2x3 confirmatory tables, to evaluate strategies and errors generated by SI. These studies have been fruitful in providing a better understanding of how students see an association between events.

Coverage of heuristics in the assessment of SI has been reported by Arkes and Harkness[14], although these studies have been fundamentally limited by the analysis of 2x2 table configuration, Batanero et al.[1], on the other hand have given some insight into the assessment of a 2x3 classification tables. Notably, among them is what is called the cell *a* approach (listed in Batenero et al.'s taxonomy[1]). A low or high frequency in the upper left hand side cell of a 2x2 classification table (i.e., "*cell a approach"*) becomes the datum for comparison to other cells in the classification tables. Because of the unusual features i.e., low or high frequency in cell *a,* the value becomes available to memory, and is easily used to compare to other cells. Research reports that it is more difficult when the cell *a* approach is used in 2x3 tables and higher. Batanero et al.[1] for instance, found that almost 14% of the respondents use the cell *a* approach to examine the association. Shaklee and Tucker[16] found a comparable 17% of their sample have used the same approach. Batanero et al.[1], by using a 2x3 and 3x3 tables the cell *a* approach diminished substantially. As these strategies effectively depend on the structure of the classification table format, they provide little information about the conception of SI. The table structure (classification tables) restricts the analysis and heuristics used for SI or associations, to strategies that evolve around context and format of tables.

From the view of the authors some of the difficulties in the assessment of SI results out of the decomposition of event into outcomes. When an event has more than two outcomes, and compared with a second event, having also two or more outcomes, the analysis of event relation is a stereotypical instance of the most salient relation. For example, in the event of alcohol intake, there are a number of outcomes that are possible, such as soberness or drunkeness, when this is related to a second event like driving an automobile; drunkenness and automobile accidents is the relation mostly called for, because it is familiar and regular, and used to assess this particular situation. Hence, in an event where alcohol intake does not lead to automobile accidents, interpretation of independence is considered as a disjunction by the two events and evokes a mental picture of "unconnectedness" and exclusivity that may mean, statistical independence. Whereas, mutual exclusive events in probability theory, explains that events are never independent. Hence, the most representative and regular feature and use of contextual and meaningful information about events and event outcomes, draws the student to make some sort of heuristic decision about SI, that is unrelated to rules and laws of SI. Research work by Smedslund[17] explained that novice students have a weak cognitive structure; subsequently a naïve understanding of the SI concept to the extent that they do not sustain the cognitive structure that helps in the empirical understanding of a relation.

Ward and Jenkins[18] and Arkes and Harkness[14] were insightful in their findings of how students use strategies in the assessment of SI. They considered that stimulus presentation and experimental conditions influence problem-solving strategies, in that a different presentation structure could call for a different heuristic for making sense of SI. The SI has significant pedagogical suppositions because introductory books in statistics develop the concept of SI through a cookbook and rule-based approach; these components lack the needed information for building strong cognitive structures for understanding SI.

Despite the extant of research on SI assessment through the use of classification tables and use of the Venn Diagrams[3], the strategies used as prospective teaching tools for SI; may lack the contextual concrete features needed in the teaching of SI. Gomez, Pozo, and Sanz[19] noted that a linear causal reasoning of analysis of problems does not challenge students with tasks out of context. Thus, by the introduction of higher ordered concepts from well-anchored ones, helps students to develop strong cognitive structures.

The Confirmation Of Statistical Independence: Statistical independence is not achieved by a sequence of events, cause and effect, or through the intersection of events[20,21]. The product of outcomes of events shows statistical independence, if and only if, these probabilities are available. If the product is equal to the joint probability of the two events "A" and "B", i.e., $P(A \cap B) = P(A \text{ and } B)= P(A) \times P(B)$, SI is then confirmed. Also, conditional probability, as a meaningful concept helps in the understanding of independence. For example, given that event A is conditioned by B, generally P(A/B) is not equal to P(A). In other words having known that B's occurrence, changes the chance of A's occurrence. In the case where P(A/B) does in fact equal to P(A). It can be said that knowledge of B's occurrence has no direct effect on A and vice-versa; thus, P(A/B)=P(A) where P(A) and P(B) can be said to be independent[23]. Algebraically, if students are able to determine conditional probabilities, they will be able to confirm statistical independence. As a general rule, statistical independence is confirmed by the multiplication of probabilities of events being related to a second, third, or *n* events. When contingency and joint data are not available, SI can be obtained from historical frequencies. Application of the product rule to confirm statistical independence is mechanic; moreover, students' may forfeit the heuristics used in the confirmation because they tend to court to a causal relation to analyze the relation between events and do so by forfeiting the statistical test. The analysis of relationships is reinforced by either a positive or negative representation of a stereotypical instance[22]. An example below illustrates the product rule of SI.

Suppose a researcher investigates smoking behavior among lung cancer cases, a researcher is seeking to find whether lung cancer is independent of smoking. The researcher has the following data available: Historically, it is known, 4% of the population has lung cancer, the probability of lung cancer is P(lung cancer)=0.04; and 12% of the population are smokers P(smokers)=0.12. It is found that 30% of all lung cancer cases are smokers. Phenomenologicaly, there might be a causal relation implied by the nature of the events and based on the frequency of occurrence or "degree of familiarity." It might be that concepts are available or heightened by the context of the problem through which, students decide whether there is a relation or not. The value for the joint probability can be calculated by the use of the probability that 30% of all lung cancer cases are smokers; and the product of the respective fractions being: $0.3 \times 0.04=0.012$ that gives the joint probability of P(lung cancer and smokers)=0.012 being not equal to the product of the probabilities, P(lung cancer) x P(smokers)=0.04 x 0.12=0.48; it is the implied that events are dependent. The important feature in this illustration is that it lends to the causal nature of a relation. Seeing relations mechanically as causal may result in lay encounters with such event as cancer and smoking behavior[24].

It is also important to differentiate with what may be considered "real data" and "gambling data." Gambling data is transient and ghostly and always changing. It can be obtained from instances where the values can be easily predicted as flipping a coin or throwing a dice. While real data can be obtained through historical frequencies. For example, the odd of a male born in the coming year is a case of historical data. There is a tendency to assess real data based on contextual features where greater attention is given to the "representativeness" that such events are accidental. Irrespective of the type of data used, assessment of SI should show whether two events are dependent or independent. In addition, independence can be thought of in degrees in the use of the Chi-square distributions and are effective in finding the significance of independence under which a certain probability of error occurs. However, this does not give a conceptual understanding of SI. In many situations events can be regularly related without being causally related. Researchers often identify causal factors distinguished from associated factors. For instance, research on the association of lung cancer and smoking has stringent criteria based on years of research that validate its legitimate causation. There is consensus at the present day, to state that such a relationship is commonly implied and usual, as it is dependent and a causal one[12]. Keeping in mind that correlation does not fetter away a causal relation and events that are correlated do not necessarily have a causal relation.

The purpose of the study is to understand how naïve students view statistical independence. Second, understanding misconceptions that are instrumental in the process of assessment of SI helps in the construction of a viable conceptual frame that provide a tutorial for the understanding of statistical independence.

METHODOLOGY

The study is divided into two parts, the first part identifies students' errors (misconceptions) of statistical independence. The second part of the study presents a teaching unit were research can be roughly classified as bridging action and evaluative research. Action research, involves participants in the research for which change is registered in a process of making change, and studying the consequences of these changes. Within this research method gaps are abridged by the accumulation of knowledge through different type of methodologies i.e., triangulation. The essential point of this type of approach is that evaluations are drawn from individuals or collective data. Specifically, interview data in this case it appears as an indicator as opposed to being a confirmation to a fact that requires checking the data through several procedures.

In the first part of the study two types of data collections were used; interview and objective multiple-choice question; in the second part of the study students were presented a tutorial followed by an informal interview. The interviews were subscribed and content analyzed as to whether change in understanding did occur. The concepts or hypothesis were examined from different approaches that provide added confirmation to justify conclusions.

Study 1*:* Forty eight university level students were recruited from statistics courses in a private university in Lebanon following the American-based semester system where English is the medium of instruction. The participants in part 1 of the study were enrolled in the Faculty of Arts and Social Sciences (25%) and the rest (75%) in the Faculty of Business Administration. The first part of the study lasted for one hour and 15 minutes. The first 15 minutes, 48 students were asked to respond to a question, based on the responses, a 45-minute group interview was conducted with 6 students based on their responses to the meaning of statistical independence. Lastly, a multiple-choice question was used to determine the type of error and the cognitive processing aspects to the misconception with all the students.

All 48 respondents were asked to provide a definition for SI. The responses to the definitions were coded and classified. Five main responses appeared, these are reported on Table 1. The 48 students were also given a multiple-choice question to gain some knowledge about the information-processing aspects of the errors. Six students of the 48, were selected for group interviews and their reactions recorded.

Study 2*:* The second part involved 12 students who came from sophomore and junior levels taking an introductory statistics course, these students did not take part in *study 1* and were interviewed a year later as part of a larger ethnographic study on students misconception in probability. The 12 students were presented with open-ended questions and probed in the form of tutorial interviews for the "action-quality" problem frame. Based on the analyses of the first part of the study the tutorial interview questions were designed accordingly.

RESULTS

Definition of SI and Interview Data: Forty-eight students were asked to provide a definition for SI. The analysis of responses showed two types of errors, these are the causal explanation and the perceptual errors (see Table 1). Eighteen students gave a causal explanation to the definition (type 2), and fourteen students gave a perceptual response (type 4). The type 2 definition was classified as the "causal framework error" and type 4 was conceptualized as the "perceptual overlap error." The causal framework error can be considered as an event which is perceived to have an effect on a dependent one, based on the analysis of context. The perceptual type of misconception (type 4) renders the relation visually as a connection or overlap of one event over another (see Fig. 2).

Based on the definition, six students were selected from the 48. The selection criteria were as follows: Three students were selected who gave definition number 2 (see Table 1) and three other students gave definition number 4. These six students then attended a lecture on probability theory, joint probability, conditional probability and SI to see if students could apply the corollaries. These six students were group interviewed for one hour and they were asked to respond to questions by writing or verbally and responses were transcribed. These six students were asked one main question "having event A and event B, how would you confirm statistical independence between the two events." Unstructured probing questions were devised to have a better understanding of students' responses.

Table 1: Students Definition of SI

Type of Response	Frequency
1. When the occurrence of one event does not change or effect the occurrence of another	2
2. When an event B dependent on A in which A is independent of B.	18
3. When there is no relation between two events	6
4. The two events are mutually exclusive	14
5. When the joint probability of the two events is the same	4
6. Non-congruent Response	4

The interview questions were used to probe into students' misconceptions. It is designated by "I" for interview questions and by "S#" for respondent(s). Those students interviewed with meaningful interpretations were included in the study:

I: How can you determine if the two events A and B are statistically independent?

S1 (made type 2 error, see Table 1): You are supposed to see if one of the events is the cause; if it is not, it is independent of the other.

S2 (made type 2 error): I have to know what the events represent [event referents e.g., A stands for gender of male, while B may stand for a success at a job] and how they relate to each other. If I do not like something it is independent of me liking it. The object is independent of me and I think it knows little of what I feel about its existence or causes and extensions [the student looks at the issue phenomenologicaly].

I: Supposse that two events are considered, being alcohol intake and driving a car, and eliminate the referent or the causes of these events. Also, if we look at the individual probabilities of each event, how would we assess if the events were independent?

S1: We have to determine the "sameness" [joint occurrence] of the events If there is something common between the events we can state they are dependent." After all those who drink [i.e., alcohol] have a greater chance of making automobile accidents. Yes, I think these events are dependent because those who drink and get drunk, could easily make accidents.

Students S2, S3 (made type 2 error), S4 (made type 4 error), and S6 (made type 4 error) agreed with S1, they easily shifted from a causal framework error to a perceptual overlap. When probed further, student contradictions were revealed:

I: So if there is a joint element between two events, does this mean being drunk causes accidents. Suppose the recent statistical report indicates that a large number of those who make automobile accidents have black hair. Does this mean that black hair is dependent on making accidents?"

Five of the students said that a joint occurrence does not mean that there is dependence. They stated that "black hair is not dependent on whether you make accidents or not." As stated by Pfannkuch and Brown[5] students may be convinced of a logical and epistemological perspective of independence that contradicts formal theories. Students were probed to see the level of awareness of their inappropriate conceptions.

I: For those who drink and do not get drunk, does that mean they could still have accidents.

S1, S2, and S4: No not necessarily.

S2: Yes, joint occurrence does not mean that the events are independent.

I: Why?

S3: like you said given that some who drink may not be susceptible to being drunk. In fact, those who do not drink could have accidents too.

Students realized that one of the events (i.e., alcohol intake) could have different outcomes, which altered the heuristic of causation i.e., those who are not drunk are not susceptible to making accidents. Another student said: "those who drink could be sober but not drunk."

S1: The condition that drinking may get you drunk is arguable and those who drink could be sober and have no accidents at all.

I: Does this mean that drinking and accidents are statistically independent?

S3: Those who crash must be drunk; hence, accidents are dependent on a lot of drinking.

Respondents realized that one of the two outcomes is functionary and causal, they then changed their conception of event causality by isolating the causes from effects, through a positive analysis i.e., representativeness or what may be considered as related outcomes of the two events. These conceptions are represented pictorially in Fig. 1. On the one hand, students see an image of two overlapping sets, or a shared event as a case of interaction and a relation which they reason through as a case of dependence. On the other hand, the causal framework is perceived to have a direct connection between events for which one event influences a second; hence, students shift between these two conceptions (perceptual overlap and causal relation) in what Clement[25] called "shifting between approaches." It is not clear if they are consciously aware of the inconsistencies or shifts, but do so because misconceptions are deeply anchored in cognition that they alternate mechanically between both type of conceptions.

What is called for first-- the causal framework or perceptual error?: The first part of the study showed that students' use of the causal explanation to analyze relationships. It appeared that it was more natural to use this type of reasoning or to follow the standard sequence from causes to consequences such that if the relationship is mediated by its meaning, it becomes very intuitive and automatic.

There are two accounts of misconceptions revealed in the above interview statements. First, there are obvious attempts at assessment based on the familiarity of the context of the relation, if the event meaning mediate a relationship, it provides the logic for a causal thinking and events are evaluated as being dependent. Second, there may be a tendency to use maxim like beliefs i.e., beliefs that are current, evident and salient to memory Konold et. al.[6], such that students assess SI through a pictographic heuristics they see events in discrete parts, believing that unshared elements produced are causes of exclusivity and hence independence. These two conceptions are illustrated in Fig. 1.

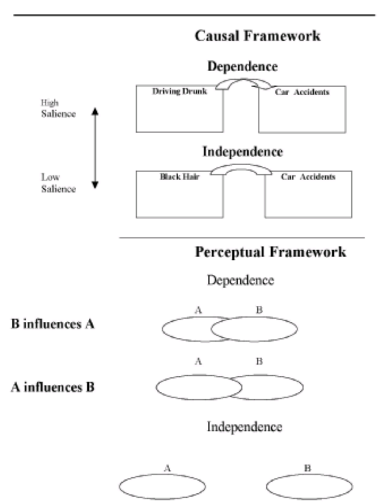

Fig. 1: A Pictographic Representation of the Two Types of Misconceptions Causal and Perceptual Framework

Based on the analysis of the interviews a multiple-choice question was administered to the same group (n=48). The multiple-choice question seeks to identify whether students were more prone to give a "causal or perceptual" response. The students were asked to select the most appropriate response to a number of statements. Three types of responses were presented to students: a "causal," "jointness", and "exclusive event". The multiple choice question is shown bwlow.

Multiple Choice Question: Which of the responses give the closest response to a correct answer?

Two events A and B are statistically independent then

a) A and B are mutually exclusive where no elements of A are found in B and no elements of B are found in A.
b) When event A causes no direct effect on B and B causes no direct effect on A.
c) When there are some commonalties of A and B meaning, there are some elements of A in B and some elements of B in A.

Eighty six percent of the students (n=38) responded to choice b and almost 14% (n=6) to choice c. There was no wrong response (choice *a).* Most of the students attributed independence to being causal. This finding suggests that the causal relation is at the front-end of the information-processing schema; meaning the causal schema is used first and stimulated by features inherit in the situation, which is common, and regular, and at the front-end of cognition.

In general, the findings showed that SI assessment is incompatible with the logic of empirical formulation. In an attempt to correct this situation, a concept-mediated tutorial was used to explore how students used their conceptual notions of SI.

Tutorial Method To Understanding SI: Based on the finding of the first study a pedagogical strategy was thought to build on previous knowledge of SI. The action-quality problem frame was used to conceptually mediate the concept of SI. Formal rules in probability and rules of algebra, were used to show independence for two related events. The action-quality problem frame went as follows:

The action variable has two outcomes (base-rates) either Rejection given by (R) or Acceptance given by (A). The quality of the product is either Good (G) or Bad (B). Inspectors accept a certain level of products and reject the rest. Historically, the inspectors found that at a certain level, all products are good, the rest are bad.

This problem frame was presented to 12 students who were not part of the first study. It goes as follows:

The first probe question started as follows:
"What is a correct decision by an inspector, meaning what action he/she takes that is considered appropriate?"

One student explained that appropriate judgments would be for the inspector to accept a good product. Only two students produced a fully correct response, to accept a good product or reject a bad product. Students were then asked the converse:
"What is a wrong decision that could be made by the inspector i.e., what action he/she takes that is considered inappropriate?"

All students agreed that an inappropriate decision would be the rejection of a good product or the acceptance of a bad product. The approach was to complement deterministic reasoning with probabilistic reasoning through the use of the data. As a sequence to the questions, the following was asked:
Suppose that inappropriate judgments are independent of appropriate ones and that there is an equal chance for each of the events. If this is our initial assumption the proof can be worked out fairly simply.

Six of the 12 students who were asked to state if a bad judgment is independent of good judgment, said it did not. Two students stated the probabilities should be equally likely and one stated that they should be equal, the rest did not respond. Those students who said "there is an equal chance for the probabilities." They explained that the appropriate judgment of product x by inspector z is completely independent of making inappropriate judgments of product y by inspector s, such that the latter does not determine the probability of the former, it is merely incidental or random..

In the next step, the merge of mathematical rules with the expectation that generalization of independence could be implied. Mathematically, two events A and B are independent if the joint probability of the two events is equivalent to the product of the probability of each event i.e., $P(A \cap B)=P(A) \times P(B)$. Using rules of algebra and probability the proof follows:
The probability of an inspector making anppropriate judgment is given by adding the probabilities of accepting a good product $P(A \cap G)$ plus rejecting a Bad product $P(R \cap B)$:

$$P(A \cap G)+P(R \cap B) \quad (1)$$

The probability of another inspector making an inappropriate judgment is given by the probabilities of rejecting a good product $P(R \cap G)$ plus accepting a bad product $P(A \cap B)$:

$P((R\cap G)+P(A\cap B)$ (2)

With the assumption that judgments are independent suggests that the probability of making a good judgment is not biased by a bad judgment, hence each event has an equal chance that explains decisions about the quality of the product are independent of each other and have an equal probabilities:

$P(A\cap G)+P(R\cap B) = P(R\cap G)+P(A\cap B)$ (3)

This assumption pertains to independence and if the multiplication rules is applied for independent events, the following can be obtained:

$[P(A)xP(G)]+ [P(R)xP(B)] = [P(R)xP(G)] + [P(A)xP(B)]$ (4)

Bringing everything to the left side and associating like terms:

$[P(A)xP(G)] - [P(A)xP(B)]+[P(R)xP(B)]- [P((R)xP(G)]= 0$ (5)

Factoring out both outcomes P(A) and P(R) for the action event:

$P(A)x[P(G)-P(B)]+ P(R)x[P(B)-P(G)]= 0$ (6)

Using subtractive associative rule: $[P(B)-P(G)]= -1x[P(G)-P(B)]$, then

$P(A)x[P(G)-P(B)]- P(R)x[P(G)-P(B)]= 0$ (7)

Factoring out $[P(G)-P(B)]$, the following is obtained:

$[P(A)-P(R)]x[P(G)-P(B)]=0$ (8)

Solving for P(A), P(R), P(G) and P(B) respectively:

$P(A)=P(R)$ given that $P(G)\neq P(B)$ and $P(G)=P(B)$ given that $P(A)\neq P(R)$

From probability theory it is known that the addition of all exhaustive outcomes of an event is 1, then $P(A)+P(R)=1$ and $P(G)+P(B)=1$, given $P(G)\neq P(B)$ and $P(A)\neq P(R)$ respectively; hence, $P(A)=0.5$, $P(R)=0.5$ and $P(G)=0.5$ $P(B)=0.5$.

It is concluded that given the assumption that events are independent, the use of statistical and algebra helped to show that independence can be conflated with “maxim-like ideas” of independence i.e., ideas that are regular and salient. Proofs are also a richer context for the understanding of abstract concepts as SI.

Several questions were raised by students to enable them to resolve misunderstanding about relations: The researcher stated; “if the marginal probabilities are equal for good and bad, or accepted and rejected events, then the conditional probabilities in any combinatorial form should be the same e.g., P(bad/rejected)=P(rejected/bad). Students then contended, how is it possible to have a probability that is bad given it is rejected. Three students were not convinced that a diagnostic relation was possible as that compared with an expected relation e.g., rejected because it is bad. Students thought that independence to be tested would have to be in the context of a causal relation. Five students agreed with the latter that there is more chance to reject given it is bad, than bad, given it is rejected. Corroborating with Tversky and Kahneman[22] results, a causal relation is always stronger than a diagnostic one. Some students negated their old conceptions and made reference back to the proof suggesting mathematically it can be proven, that independence of events should reflect equal conditional probabilities of the causal type P(rejected/bad) and the diagnostic type (viz., P(bad/rejected)). Although students were receptive to the proof they went back to their old and lay conceptions.

REVIEW OF TUTORIAL PROBLEM

The plethoras of studies have covered the difficulty in conceptual change[26, 27] suggest that change may be difficult to establish in the remediation. Rather than change misconceptions they build from them. The aim of the interview tutorial was to conceptually mediate SI, through the experience with real world context . Konold[28] suggested that students be treated as scientists where data are available, and mechanism of discovery is in place for higher ordered thinking. Pfannkuch and Brown[5] explained that mathematical modeling, and assumptions, must be made explicit when real data are explored for constructive remediation. The method in the second part of the study used specific notions that help students’ to move away from deterministic reasoning. The goal was to have students become aware of their own thinking through a situation or context which is familiar and regular.

When formal proof was provided to students, they alluded to old conceptions and imposed a causal relation to assess independence. Students used common sense notions about independence and dependence. The presentation of the proof helped the students to switch from deterministic reasoning to probabilistic reasoning, and model the concepts mathematically. In addition, students were able to see causality depending on the probabilities of chance of each event not whether they are incidental, causal, or a diagnostic one.

DISCUSSION

The interview questions revealed that students were making decisions for SI as a criteria that were intuitive. Particularly, interview data showed students conceptual shift from a causal schema to a perceptual overlap. Those students, who received formal instruction in the assessment of SI, often turned to old, lay and intuitive conceptions of SI. Even those who gave correct responses, were rule-based and rote as opposed to being meaningfully mediated.

Additional probes into students thinking revealed a number of inconsistencies and insights into SI assessment. Three important findings were evident as a result of the first investigation. First, students viewed independence in the framework of a causal relationship. Second, students perceived events having one possible outcome as opposed to having two or more outcomes (base-rates). When it was evident that events could have more than one outcome, this changed the perception about the relation between the events. What was apparent is that students used the most representative conception (i.e., people estimate where the occurrence of an event is based on similar features of a highly possible interaction between two events). For example, students believe there is more chance that drunkenness and making accidents rather than being sober, and making accidents. This possibility of event relation created a conflict in students' conceptions when they tried to resolve the inconsistency, it negated their initial knowledge structure. The attempt was to integrate past conception with new ones but, experienced some form of resistance that forced them to return to their old conception. The literature potently agrees that tightly held conceptions are not easy to change, even after proper instruction and formal learning. Individuals provide inappropriate responses to various data forms that pedagogically could be illustrated through a classification table format. Students encounter notions of relations in form of matrices at an early age in schooling. Developmentally, however, when ever data is presented in other forms it engenders their conception of SI. Consequently, they try to make sense of relations by imposing a contingency structure, if the superimposed structure is incomplete; they alternate to a causal schema for interpretation and assessment.

The seminal research on the psychology of probabilistic thinking by Tversky and Kahneman[4] has had an interest in the idea of decision theory. Those researchers among others like Jenkins & Ward[15]; Shaklee & Tucker[16]; Batanero, et., al.[7] have looked at association from a heuristic orientation, the research literature gave an indication that substantial pedagogical activities might be appropriate for the development of SI assessment and that the calculation of SI must be merged with logical interpretation, as this study has shown. In general the finding report that students in introductory statistics courses look for representative features that are context bound to decide for SI. In cases where the problem context or the question varies in the structure, it overshadows the context, especially where the structure of the problem opens up to different possible heuristic choices, or to other problem solving approaches that are requisite to the choice of the heuristic used.

The use of the action-quality problem frame provided students the opportunity to make logical judgments about SI. For example, the acceptance of data-logical information of good items has greater chance of acceptance than bad items. As Kelly and Zweirs[12] explained that posterior knowledge about events biases student judgment about the succeeding events. Similarly, Konold[28] demonstrates an example in a coin tossing experiment in which a fair coin in sequence is tossed six times. Students predict the occurrence of the last three tosses of six throws, through the knowledge of the outcome of the first three throws. This in turn gives some knowledge about the likelihood of outcomes of the n^{th} event, which decreases or increases the probability of the previous n probabilities depending on the posterior outcomes. In fact, if the coin is unbiased, the probability of each outcome is equally likely, meaning if the coin is fair a toss at any sequence is independent of the other and have an equal probability; however, the correlation between two events has different conceptual rules than that of the concept of independence of likely outcomes, repeated over n times. Given some posterior knowledge about the failure engenders the assessment of independence for the consequent outcomes. This difference can be characterized in the use of multiplication rules to explain Konold's *coin tossing problem* that proves independence; comparatively the conditional probability is used to prove independence when relating two event outcomes.

This study also argues with Konold's[28], findings, that independence or "zero-correlation" is a well-developed concept prior to instruction. In fact, Konold[29] viewed independence as temporal; that is, individuals are given some knowledge about the outcomes of the experiments, which provide the perceived likelihood of other subsequent events. For instance, if a coin is tossed five times, the occurrence of three tails, on the first, second, and third toss gives the perception that posterior events bias the expectations of the subsequent throws or events, being either a head or tail[2]. Decision whether there is statistical independence is quite different. SI; is epistemologically different, and may be viewed as a relation being causal, in some sort of an imagined context.

If it is true that the assessment of independence has two interpretations: The conditional probability being conditioned by an independent event, $P(A/B)=P(A)$ or through the application of the multiplication rule, $P(A \cap B)=P(A)xP(B)$. It is probably more logical and interpretable to make sense of the conditioned event being independent of the conditioner, which helps to consider the probability equal to the conditioned event as a more meaningful interpretation for independence. It is imperative that a conceptual understanding of joint probability is fundamental and key to understanding the procedure of SI assessment. It is also found that judgments of each of the probabilities conflicted with the calculus of SI and hence lead to an inappropriate assessment.

Even with the use of rule-based formulas the concept of SI is not well anchored in students' cognition. Rules of SI are loosely connected and vulnerable to change depending on the situation. Pfannkuch & Brown[5] and Sperber[30] explained that intuitions are meshed into the learning process they become spontaneous after repetitive exercise, to the extent of becoming by-products of formal instruction or part of students personal experience and data-logical frames. Consequently, understanding differences between causality and correlation are fundamental prerequisites to understanding the concept of SI.

While students strongly hold to beliefs and intuitions conflicting with formal theories. It is a substantive task for teachers to select and judge whether the material or teaching method is appropriate in building new intuitions consistent with formal structures. When context is introduced as to make decision if the events are independent or not, students have a tendency to alternate from one conception to another. In essence, the very nature of statistics and probability is contrary to the traditional deterministic approaches[9]. In addition, students attempt to search for a causal relation because of their epistemological precepts about the analysis of relations. Evidence from our interviews suggests a shift from probabilistic to deterministic reasoning, is key for teachers to use these instances to ferment the meaning to such an abstract concept as SI.

CONCLUSION AND RECOMMENDATIONS

Many students gave a general and lay definition of SI that did not match with the true statistical interpretation. Even when students were probed, they gave an incorrect response. It is apparent that students make spontaneous intuitions from lay and deep-seated views that interfere with the learning of new concepts. This generally opens up many avenues for researchers and teachers to remediate and provide proper instructional procedure for SI assessment.

The following recommendation should offer aid to those teachers and researchers in the field of statistics and decision theory:

1. From this study and key to what can be done to help students approach statistical concepts like SI, it is suggested that students be presented in structured and hierarchical order; intersection; conditional probability; and finally SI rules in sequence. It is realized that many books are following this sequence, but it is apparent the approach is a cook-book one— a need for a conceptual mediated method would furnish a more meaningful approach were students can anchor concepts firmly in their cognitive structures.
2. When students are questioned they should be able to provide feedback through probing questions to make students aware of their inconsistencies[31]. If possible, such correction will be followed by an exploratory algebraic proof that counters their intuitive understanding of concepts.
3. Formally students should be pressed to differentiate between probabilistic thinking and deterministic thinking before starting a unit on probability[5]. In addition, teachers should be able to have students differentiate between probabilistic or deterministic thinking and be able to differentiate between both when solving problems.
4. Using pictorial presentations or figures could help students see perceptually in order to conceptualize the problem of SI. The different representational systems for the same concept can help students see the problem of SI from different perspectives,

either through the classification table, tree diagram or chart like graph.

5. Despite the fact that some research in decision theory has aimed at studying student conception of independence through the use of classification tables or through the use of Venn Diagrams[3], these perceptual methods fall short of a fully developmental model to account for SI assessment. It further draws the researchers to extend their research and call others, in an effort to provide better teaching methods for SI assessment.

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