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Free Convection over a Permeable Horizontal Flat Plate Embedded in a Porous Medium with Radiation Effects and Mixed Thermal Boundary Conditions

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Abstract: Problem statement: In this study, the mathematical modeling of free convection boundary layer flow over a permeable horizontal flat plate embedded in a porous medium under mixed thermal boundary conditions and radiation effects is considered. **Approach:** The transformed boundary layer equations are solved numerically using the shooting method. **Results:** Numerical solutions are obtained for the wall temperature, the heat transfer coefficient, as well as the velocity and temperature profiles. The features of the flow and heat transfer characteristics for different values of the radiation parameter N_R, the mixed thermal boundary condition parameter ε and the suction or injection parameter γ are analyzed and discussed. **Conclusion:** We can conclude that when the suction or injection parameter γ is fixed, an increase in the radiation parameter N_R leads to the increase of $-\theta'(0)$ (for the case of constant surface temperature). However, when γ is fixed, an increase in N_R leads to the decrease of $\theta(0)$ (for both cases of constant surface heat flux and mixed thermal boundary conditions).

Key words: Free convection, mixed thermal boundary conditions, permeable horizontal flat plate, porous medium, radiation effect

INTRODUCTION

The transport properties of fluid-saturated porous materials are due to the increasing number of important applications in many modern industries, ranging from heat removal processes in engineering technology and geophysical problems. Further examples of convection through porous media may be found in manmade systems such as granular insulations, winding structures for high-power density electric machines and the cores of nuclear reactor (Bejan, 2004; Pop and Ingham 2001; Nield *et al*., 2006).

Following Pop and Ingham (2001) and Ping and I-Dee (1976) were probably the first to consider the similarity solutions for the free convection boundary layer flow about a heated horizontal impermeable surface embedded in a porous medium, where the surface temperature is a power function of the distance from the origin. In a subsequent paper, Chang and Cheng (1983) pointed out that this boundary layer approximation is identical to the governing equations for the first order inner problem in a matched asymptotic expansion in which other effects, such as fluid entrainment, were taken into consideration. The free convection over an impermeable horizontal flat plate embedded in a porous medium has been discussed by many researchers such as Hsu *et al*. (1978); Ingham *et al*. (1985); Nazar *et al*. (2006) and Moghaddam *et al*. (2009). For permeable surfaces, Chaudhary *et al*. (1996) studied the natural convection from a horizontal permeable surface in a porous medium. Results obtained from various asymptotic analyses are found to compare well with those obtained from the direct numerical integration of the equations.

Radiation effects on free convection flow are important in the context of space technology and processes involving high temperature and very little is known about the effects of radiation on the boundary-layer flow of a radiating fluid past a body. The inclusion of radiation effects in the energy equation, however, leads to a highly nonlinear partial differential equation (Hossain *et al*., 2001). Hossain *et al*. (1999;

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2001) have studied the effects of radiation on the free convection from a porous vertical plate with a uniform surface temperature, a uniform rate of suction and with variable viscosity, respectively. Recently, Bataller, (2008; Cortell 2008) separately studied radiation effects on Blasius and Sakiadis flows when the plate is maintained at a constant temperature and conjugate boundary conditions, respectively. He determined the effects of physical parameters like Prandtl number Pr and radiation parameter N_R on the heat transfer characteristics. The previous studies of the free convection boundary layer flow over horizontal surfaces embedded in a porous medium dealt with numerical solutions associated with either prescribed/constant surface temperature or heat flux. Ramanaiah and Malarvizhi (1992) were the first to consider free convection adjacent to a wedge and a cone that are subjected to mixed thermal boundary conditions. On the other hand, Nazar *et al*. (2006) studied the free convection boundary layer flow over vertical and horizontal surfaces in a porous medium with mixed thermal boundary conditions. Laminar free convection flows about a wedge, a cone and a vertical spinning cone under mixed thermal boundary conditions and magnetic field were solved numerically using the Thomas algorithm by Ece (2005); Ece and Ozturk, 2009), respectively.

MATERIALS AND METHODS

The aim of the study is to investigate the radiation effects on free convection boundary layer flow over a horizontal flat plate embedded in a porous medium under mixed thermal boundary conditions.

Analysis: Consider the steady free convection from a horizontal flat plate embedded in a fluid-saturated porous medium of uniform temperature T_∞. Dimensional coordinates are used with $\bar{x}$-axis measured along the surface and $\bar{y}$-axis being normal to it. The boundary layer equations which govern the steady free convection flow over a horizontal surface which is embedded in a fluid-saturated porous medium are of the following form (Pop and Ingham, 2001):

$$\frac{\partial \bar{u}}{\partial \bar{x}}+\frac{\partial \bar{v}}{\partial \bar{y}}=0 \tag{1}$$

$$\frac{\partial \bar{u}}{\partial \bar{y}}=-\frac{gK\beta}{\nu}\frac{\partial T}{\partial \bar{x}} \tag{2}$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}}+v\frac{\partial T}{\partial \bar{y}}=\alpha_m\frac{\partial^2 T}{\partial \bar{y}^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial \bar{y}} \tag{3}$$

where, $\bar{x}$ and $\bar{y}$ are the Cartesian coordinates with $\bar{x}$-axis measured along the horizontal plate and $\bar{y}$-axis is the coordinate normal to the plate, v is the kinematic viscosity, g is the gravitational acceleration, β is the thermal expansion coefficient of the fluid, K is the permeability of the saturated porous medium, T is the temperature across the thermal boundary layer, T_∞ is the constant temperature of ambient fluid, $\alpha_m = k/\rho c_p$ is the effective thermal diffusivity, k is the thermal conductivity, ρ is the fluid density and c_p is the specific heat of the fluid at constant pressure.

We shall solve Eq. 1-3 assuming that the boundary conditions for the velocity components $\bar{u}$ and $\bar{y}$ along with the mixed thermal and concentration boundary conditions are Eq. 4:

$$\begin{aligned}&\bar{v}(\bar{x},0)=v_w(\bar{x})\\&A(\bar{x})T(\bar{x},0)-B(\bar{x})\left.\frac{\partial T}{\partial \bar{y}}\right|_{\bar{y}=0}=C_1(\bar{x})\\&T(\bar{x},\bar{y})\to T_\infty,\quad \text{as}\quad \bar{y}\to\infty\end{aligned} \tag{4}$$

where, $\bar{v}_w(\bar{x})$ is the mass transfer velocity with $\bar{v}_w(\bar{x})>0$ when fluid is injected into the flow from the wall and $\bar{v}_w(\bar{x})<0$ when fluid is removed through the wall. Let $A(\bar{x})$, $B(\bar{x})$ and $C(\bar{x})$ be the undetermined functions of $\bar{x}$.

Using the Rosseland approximation for radiation (Bataller, 2008), the radiative heat flux is simplified as:

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial \bar{y}} \tag{5}$$

where, σ* and k* are the Stefan-Boltzmann constant and the mean absorption coefficient, respectively. We assume that the temperature differences within the flow through the porous medium such as that the term T^4 may be expressed as a linear function of temperature. Hence, expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, we get:

$$T^4 \cong 4T_\infty^{\,3}\,T-3T_\infty^{\,4} \tag{6}$$

In view of Eq. 3, 5 and 6 reduces to:

$$\bar{u}\frac{\partial T}{\partial \bar{x}}+\bar{v}\frac{\partial T}{\partial \bar{y}}=\left(\alpha_m+\frac{16\sigma^* T_\infty^{\,3}}{3\rho c_p k^*}\right)\frac{\partial^2 T}{\partial \bar{y}^2} \tag{7}$$

From this equation it is seen that the effect of radiation is to enhance the thermal diffusivity. Following Bataller (2008), we take $N_R = kk^*/(4\sigma^* T_\infty^{\,3})$ as the radiation parameter, so that Eq. 7 becomes:

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \frac{\alpha_m}{k_0}\frac{\partial^2 T}{\partial \bar{y}^2} \quad (8)$$

where the dimensionless parameter k_0 is defined as:

$$k_0 = \frac{3N_R}{3N_R + 4} \quad (9)$$

Further, we introduce now, the following dimensionless variables Eq. 9 and 10:

$$\begin{aligned} & x = \bar{x}/L, \quad y = Ra^{1/3}(\bar{y}/L) \\ & u = Ra^{-2/3}(L/\alpha_m)\bar{u} \\ & v = Ra^{-1/3}(L/\alpha_m)\bar{v} \\ & \theta = (T - T_\infty)/(T_r - T_\infty) \end{aligned} \quad (10)$$

where, T_r is the reference temperature and $Ra = gK\beta(T_r - T_\infty)L/\alpha_m \nu$ is the Rayleigh number for a porous medium Thus, Eq. 1, 2 and 8 can be written as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (11)$$

$$\frac{\partial u}{\partial y} = -\frac{\partial \theta}{\partial x} \quad (12)$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{k_0}\frac{\partial^2 \theta}{\partial y^2} \quad (13)$$

And the boundary conditions (4) become:

$$\begin{aligned} & v(x,0) = v_w(x) \\ & A(x)(T_r - T_\infty)\theta(x,0) \\ & \quad - B(x)(T_r - T_\infty)\frac{Ra^{1/3}}{L}\left.\frac{\partial \theta}{\partial y}\right|_{y=0} = C(x) \\ & \theta(x,y) \to 0 \quad \text{as} \quad y \to \infty \end{aligned} \quad (14)$$

We look for a similarity solution of Eq. 11-15 of the following form:

$$\begin{aligned} & \psi = x^{1/3} f(\eta), \quad \theta = \theta(\eta) \\ & \eta = y / x^{2/3} \end{aligned} \quad (15)$$

where, Ψ is the stream function defined in the usual way as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$, which identically satisfy Eq. 11. In order that similarity solution exists, we assume that $v_w(x) = (2/3)\, x^{-1/3}\gamma$, where $\gamma > 0$ is for injection and $\gamma > 0$ is for withdrawal of fluid. Substituting 15 into Eq. 12 and 13, we obtain the following ordinary differential equations:

$$f'' - \frac{2}{3}\eta\,\theta' = 0 \quad (16)$$

$$\theta'' + \frac{1}{3}k_0 f\,\theta' = 0 \quad (17)$$

where primes denote differentiation with respect to η. It is worth mentioning here that when $k_0 = 1$, the thermal radiation effect is not considered. The boundary conditions (14) reduce to Eq. 18 and 19:

$$\begin{aligned} & f(0) = -\gamma \\ & a(x)(T_r - T_\infty)\theta(0) - b(x)(T_r - T_\infty)^{4/3}\theta'(0) = 1 \\ & f'(\infty) \to 0, \quad \theta(\infty) \to 0 \quad \text{as} \quad \eta \to \infty \end{aligned} \quad (18)$$

Where:

$$\begin{aligned} & a(x) = \frac{A(x)}{C(x)} \\ & b(x) = \frac{B(x)}{C(x)}\, x^{-2/3}\left(\frac{gK\beta}{\alpha_m \nu L}\right)^{1/3} \end{aligned} \quad (19)$$

Each of these functions a(x) and b(x) must be equal to a constant to enable a similarity solution. For given values of the constants a, b and T_∞, the reference temperature T_r may be chosen to satisfy the following equation without any loss of generality Eq. 20:

$$a(T_r - T_\infty) + b(T_r - T_\infty)^{4/3} = 1 \quad (20)$$

By defining $\xi = b(T_r - T_\infty)^{4/3}$, the thermal boundary conditions (18) can be written as Eq. 21:

$$(1-\xi)\theta(0) - \xi\theta'(0) = 1 \quad (21)$$

It is worth noticing that the case $\xi = 0$ corresponds to the constant surface temperature (CWT) θ (0) = 1, the case $\xi = 1$ corresponds to the constant surface heat flux (CHF) θ' (0) = -1 and the case $\xi = \infty$ corresponds to the mixed thermal boundary conditions (MBC).

RESULTS AND DISCUSSION

Equation 16 and 17 subject to boundary conditions 18 are solved numerically using the shooting method for the cases of CWT when $\xi = 0$, CHF when $\xi = 1$ and MBC when $\xi = \infty$. Values of γ considered are $\gamma = -1 < 0$ (suction), $\gamma = 0$ (impermeable wall) and $\gamma = 1 > 0$ (injection).

Table 1: Values of $-\theta'(0)$ when $\gamma = 0$, $k_0 = 1$ (the thermal radiation effect is not considered) and $\xi = 0$ (CWT case)

$\xi = 0$ (CWT) $-\theta'(0)$ s	
Chang and Cheng (1983)	Present
0.4299	0.4300

Table 2: Values of $-\theta'(0)$ for various values of N_R when $\xi = 0$ (CWT) and $\gamma = -1$, 0 and 1

N_R	$\gamma = -1$ (suction)	$\gamma = 0$ (impermeable wall)	$\gamma = 1$ (injection)
0.1	0.1887	0.1766	0.1656
0.5	0.3267	0.2786	0.2360
1	0.4008	0.3242	0.2584
3	0.5074	0.3798	0.2773
7	0.5619	0.4053	0.2824
10	0.5771	0.4121	0.2834
100	0.6138	0.4278	0.2854
1000	0.6180	0.4295	0.2855

Table 3: Values of $\theta(0)$ for various values of N_R when $\xi = 1$ (CHF) and $\gamma = -1$, 0 and 1

N_R	$\gamma = -1$ (suction)	$\gamma = 0$ (impermeable wall)	$\gamma = 1$ (injection)
0.1	3.5514	3.6668	3.7826
0.5	2.3831	2.6061	2.8434
1	2.0525	2.3286	2.6295
3	1.7220	2.0646	2.4579
7	1.5936	1.9669	2.4038
10	1.5613	1.9429	2.3917
100	1.4887	1.8891	2.3667
1000	1.4810	1.8835	2.3642

Table 4: Values of $\theta(0)$ for various values of N_R when $\xi = \infty$ (MBC) and $\gamma = -1$, 0 and 1

N_R	$\gamma = -1$ (suction)	$\gamma = 0$ (impermeable wall)	$\gamma = 1$ (injection)
0.1	173.8079	180.0499	186.3511
0.5	39.9806	46.0506	52.4915
1	23.3410	29.3049	35.8520
3	12.3566	18.1411	24.8680
7	9.2675	14.9515	21.7791
10	8.5786	14.1338	21.0904
100	7.1427	12.7267	19.5120
1000	7.0000	12.5760	19.5120

Table 1 shows the numerical values of $-\theta'(0)$ when $\xi = 0$ (CWT case) with $\gamma = 0$ and $k_0 = 1$, where the thermal radiation effect is not considered.

Table 5: Values of $f'(0)$ for various values of N_R when $\xi = \infty$ (MBC) and $\gamma = -1$, 0 and 1

N_R	$\gamma = -1$ (suction)	$\gamma = 0$ (impermeable wall)	$\gamma = 1$ (injection)
0.1	79.4118	81.7598	84.1261
0.5	18.5942	20.9153	23.3079
1	11.0096	13.3097	15.7222
3	5.9752	8.2394	10.6853
7	4.5470	6.7907	9.2551
10	4.2269	6.4647	8.9344
100	3.5571	5.7802	8.2628
1000	3.4903	5.7118	8.1958

The numerical result obtained by an implicit finite-difference scheme as reported by Chang and Cheng (1983) for the case of CWT is included in this table for comparison purposes. It is found that the agreement between the previously published results with the present one is very good. We can conclude that this numerical method works efficiently for the present problem and we are also confident that the results presented here are accurate.

Values of $-\theta'(0)$ for various values of N_R when $\xi = 0$ (for the case of CWT) and $\gamma = -1$, 0 and 1 are presented in Table 2. It is observed that $-\theta'(0)$ increases with increasing the radiation parameter N_R for all cases of $\gamma = -1$, 0 and 1. When N_R is fixed, it can be seen that $-\theta'(0)$ is higher for and $\gamma = -1 <0$ (suction) rather than those for $\gamma = 0$ and $\gamma = 1>0$ (injection).

Table 3 presents the values of $\theta(0)$ for various values of N_R when $\xi = 1$ (for the case of CHF) and $\gamma = -1$, 0 and 1. It is found that $\theta(0)$ decreases with increasing the radiation parameter N_R for all cases of $\gamma = -1$, 0 and 1. For fixed N_R, it can be seen that $\theta(0)$ is lower for $\gamma = -1<0$ (suction) rather than those for $\gamma = 0$ and $\gamma = 1 >0$ (injection).

Table 4 and 5 present the values of $\theta(0)$ and $f'(0)$, respectively, for various values of N_R when $\xi = \infty$ (MBC) and $\gamma = -1$, 0 and 1. It is found that both $\theta(0)$ and $f'(0)$ decrease with increasing the radiation parameter N_R for all cases of $\gamma = -1$, 0 and 1. For fixed N_R, it can be seen that $\theta(0)$ and $f'(0)$ is lower for $\gamma = -1 <0$ (suction) rather than those for $\gamma = 0$ and $\gamma = 1 <0$ (injection).

The trend for MBC case is similar to the CHF case but different from the CWT case. It is worth mentioning that the numerical values given in Table 1-5 are very important and they serve as a reference against which other exact or approximate solutions can be compared in the future.

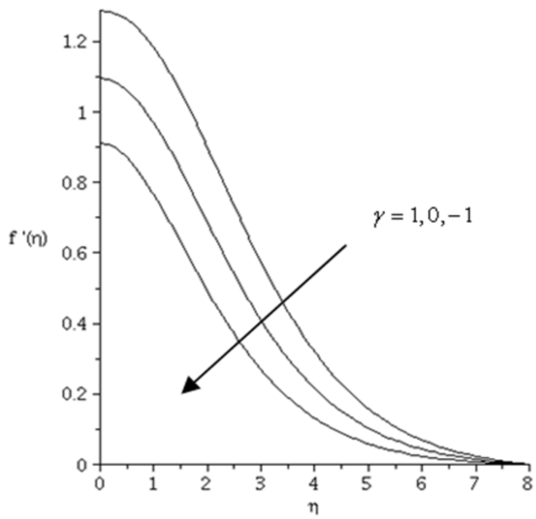

Fig. 1: Velocity profiles when and (CWT case)

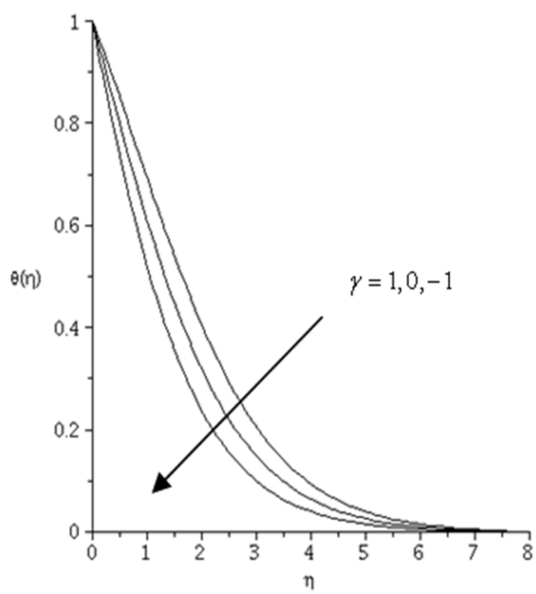

Fig. 2: Temperature profiles when and (CWT case)

Figure 1-6 illustrate the velocity and temperature profiles when $N_R = 10$, $\xi = 0$ (CWT), $\xi = 1$ (CHF) and $\xi = \infty$ (MBC), respectively. These figures show that the values of f' (η) and θ(η) decrease from 1 to 0 as η increases from zero at different values of γ. Also, it is noticed that as γ decreases, both the velocity and temperature profiles decrease and also the thermal boundary layer thickness decreases.

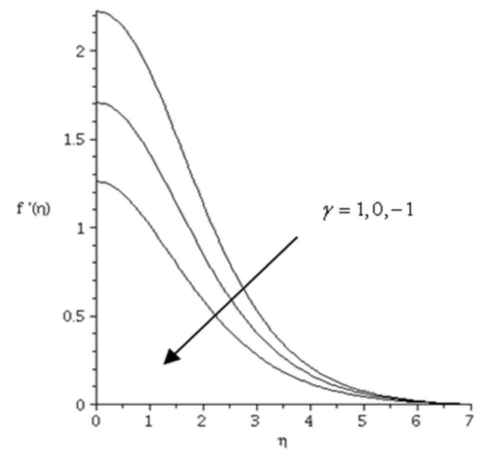

Fig. 3: Velocity profiles when and (CHF case)

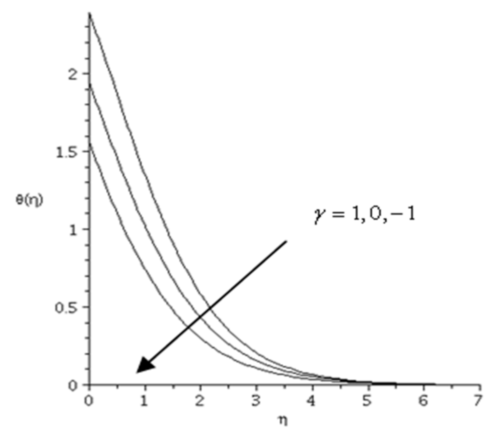

Fig. 4: Temperature profiles when and (CHF case)

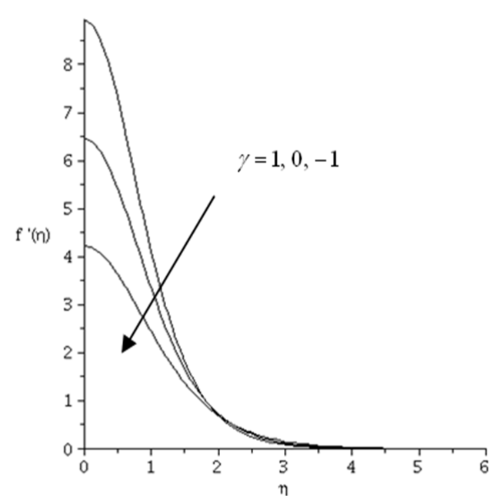

Fig. 5: Velocity profiles when NR = 10 and (MBC case)

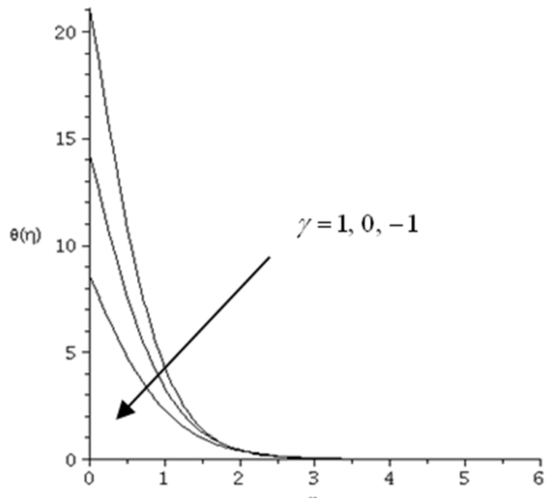

Fig. 6: Temperature profiles when and (MBC case)

CONCLUSION

In this study, we have numerically studied the problem of free convection over a permeable horizontal flat plate embedded in a porous medium under mixed thermal boundary conditions with thermal radiation effects. We can conclude that when γ is fixed, an increase in the radiation parameter N_R leads to the increase of -θ' (0) (for the case of CWT). But, when γ is fixed, an increase in the radiation parameter N_R leads to the decrease of θ (0) (for both cases of CHF and MBC).

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